Bergen County Mathematics League

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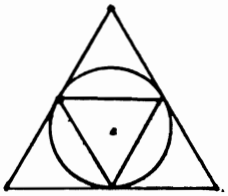
Brief Contest Solutions #1 **2007-2008**

1-1) Since $\frac{22}{7}$ is a better approximation to π than 3.14, it follows that $\frac{22}{7} \times 99 = \frac{2178}{7} = 311.14...$ is closer to the exact value than is $99 \times 3.14 = 310.86$. Therefore $99\pi > 311$, slightly, so the units' digit is $\boxed{1}$.

1-2) The ratio of tire circumferences is the same as the ratio of number of revolutions needed to cover a fixed distance. Since $\frac{26}{25.5} = 1.0196... \approx \boxed{2.0}\%$ more when we round off 1.96%.

1-3) The sum of the digits is 9, so every arrangement is divisible by 9. Therefore, the number of primes is $\boxed{0}$.

1-4) Since the diagram is self-explanatory the ratio is 4:1, so $4(12) = \boxed{48}$.



1-5) The result is easy to see by inspection. Since the sum of the square roots is 5, it's easy to guess. More formally, one can solve $\sqrt{x} + \sqrt{x+5} = 5$ to get $x = 4$ and $x+5 = 9$: $\boxed{4, 9}$.

1-6) $P(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are positive integers. $P(1) = a+b+c+d = 10$; $P(10) = 1000a + 100b + 10c + d = 1234$. From the equation $a+b+c+d = 10$, we know $1 \le a, b, c, d \le 7$. Therefore $a = 1, b = 2$, $c = 3, d = 4$ are only integers in this interval for which $P(10) = 1234$. Finally $P(3) = 1(3)^3 + 2(3)^2 + 3(3) + 4 = \boxed{58}$.