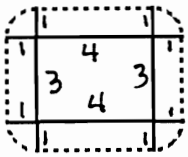
Bergen County Mathematics League

Problem Author:
Steve Conrad
www.mathleague.com

Problem Editor:
Dan Flegler
www.mathleague.com

Brief Contest Solutions #2 — **2007-2008**

2-1) 

The boundary is the dotted line. The quarter-circles have radius-length 1. The area is the sum of the areas of 4 quarter-circles + 5 rectangles = $4\left(\frac{\pi r^2}{4}\right) + 2\times(1\times4) + 2\times(1\times3) + (4\times3) = \pi + 8 + 6 + 12 = \boxed{26+\pi}$.

2-2) (#divisible by either or both) = (# div by 3) + (# divisible by 4) − (# div by 12)
$= \left[\frac{1200}{3}\right] + \left[\frac{1200}{4}\right] - \left[\frac{1200}{12}\right] = 400 + 300 - 100$
$= 600$
divisible by neither $= 1200 - 600 = \boxed{600}$.

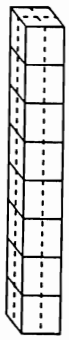
2-3) $\binom{n}{2} = 105 \iff \frac{(n)(n-1)}{(2)(1)} = 105 \iff n^2 - n - 210 = 0$
$\iff (n+14)(n-15) = 0$
$n = \boxed{15}$.

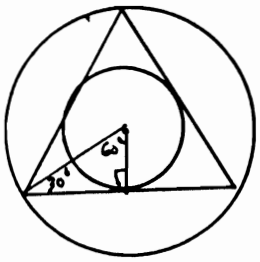
2-4) If each slice doubles the number of slices, we can create $2^6 = 64$ sliced pieces in $\boxed{6}$ slices. Can we make them all cubes?

slice 1
slice 2
slice 3

We now have 8 cubes. Make the 2 slices shown in the diagram at far right, on this line.

We now have 32 1×1×2 solids — that look like 32 bricks. Line them up so 1 more slice creates 64 cubes.



2-5) 

Ratio of radii = 2 to 1.
Ratio of areas = $\left(\frac{2}{1}\right)^2 = 4$ to 1
Area of smaller = $\frac{60}{4} = \boxed{15}$.

2-6) Each minute, distance covered by a tire = $(600)\left(\frac{5\pi}{2}\right) = 1500\pi$
In an hour, a tire covers a distance of $60 \times 1500\pi$ feet. The car's speed, in mph, is $\frac{60 \times 1500\pi \text{ feet}}{5280 \text{ feet/mile}} = \boxed{\frac{375\pi}{22}}$ mph