Bergen County Mathematics League

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Brief Contest Solutions #3

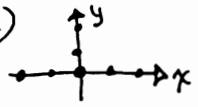
2007-2008

3-1)

D	R	T
16 km	48 km/hr	20 mins.
20 km	40 km/hr	30 mins.
24 km	36 km/hr	40 mins.

Total Time = 1.5 hrs
Total Distance = 60 km
∴ $\boxed{40}$ km/hr.

3-2)

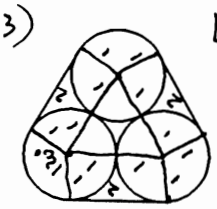


7 points

$\binom{7}{3} - \binom{5}{3} - \binom{3}{3} = 35 - 10 - 1 = \boxed{24}$.

$\binom{5}{3}$: 3-tuples all on x-axis; $\binom{3}{3}$: triples all on y-axis

3-3)



Length = circum of 3 120° arcs + 3 × 2
= circum of 1 circle + 6 = $\boxed{2\pi+6}$.

3-4) $\frac{1}{(n)(n+1)} = \frac{n+1}{(n)(n+1)} - \frac{n}{(n)(n+1)} = \frac{1}{n} - \frac{1}{n+1}$, so

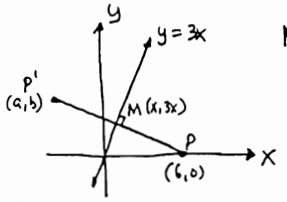
$\frac{1}{1\times2} = 1 - \frac{1}{2}$
$\frac{1}{2\times3} = \frac{1}{2} - \frac{1}{3}$
etc

$\text{Sum} = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{98} - \frac{1}{99} + \frac{1}{99} - \frac{1}{100} = 1 - \frac{1}{100} = \boxed{\frac{99}{100}}$.

3-5) $31^2 = 961$ and $32^2 = 1024$, so $1^2 - 31^2$ are the 31 perfect squares ≤ 999
$9^2 = 81$, $10^2 = 100$ so $1^2 - 9^2$ are the 9 perfect squares ≤ 99
$31 - 9 = 22$ perfect squares from 100 – 999

So prob = $\frac{22}{900} = \boxed{\frac{11}{450}}$.

3-6)



Method I:
Since $m_{\overline{MP}} = -\frac{1}{3}$, $\frac{3x-0}{x-6} = -\frac{1}{3}$, so $x = \frac{3}{5}$
$3x = \frac{9}{5}$
M is the midpt of $\overline{P'P}$. Therefore,

$\left.\begin{array}{l} \frac{a+6}{2} = \frac{3}{5} \Rightarrow a = \frac{-24}{5} \\ \frac{b+0}{2} = \frac{9}{5} \Rightarrow b = \frac{18}{5} \end{array}\right\} \Rightarrow (a,b) = \boxed{\left(\frac{-24}{5}, \frac{18}{5}\right)}$

Method II:
By ~ Δ,
$10x = 6$, so $x = 3/5$
Continue as in Method I.