Bergen County Mathematics League

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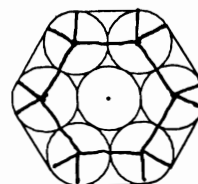**Brief Contest Solutions #4**

2007-2008

4-1) Here are all 3-digit squares whose digits are all different:
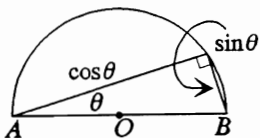169, 196, 256, 289, 324, 361, 529, 576, 625, 729, 784, 841, 961

$$\boxed{361, 529, 784}$$

4-2) length = perim of hexagon + circumf of $\odot$*
= 6(2) + 2π = $\boxed{12 + 2\pi}$.

*each arc is a 60° arc, so six of them, when their lengths are added, will equal the circumference.

4-3 $\dfrac{x^{n+2} + x^n}{x^{n+4} + x^{n+2}} = \dfrac{x^n(x^2+1)}{x^{n+2}(x^2+1)} = \dfrac{x^n}{x^{n+2}} = x^{\boxed{-2}}$.

4-4)

Area of △ = $\frac{1}{2}bh = \frac{1}{2}(\sin\theta)(\cos\theta)$; Area of semicircle = $\dfrac{\pi r^2}{2} = \dfrac{\pi}{8}$

Setting equation: Area semicircle = 2(Area of rt △)

$\dfrac{\pi}{8} = \sin\theta\cos\theta$

Thus, $2\sin\theta\cos\theta = \sin 2\theta = \dfrac{\pi}{4}$, so $2\theta = \text{Arcsin}\dfrac{\pi}{4}$, $\theta = \frac{1}{2}\text{Arcsin}\dfrac{\pi}{4}$
$\approx \boxed{26°}$

4-5) $(x!)^2 - 7x! + 6 = (x! - 6)(x! - 1) = 0$

$x! = 6$ $x! = 1$
$x = 3$ $x = 0, 1$ $\boxed{0, 1, 3}$

4-6) The least such number is 1023. How many numbers begin $10\underline{?}\underline{?}$? Well, $10\underline{a}\underline{b}$ has 8 ways to choose "a" and 7 to choose "b" ⇒ 56 such numbers begin with $10\underline{?}\underline{?}$. Now count $1203 \to 1298$, so 112 so far. Thus, the 111th entry is $\boxed{1297}$.

$\underbrace{1203 \to 1298}_{56 \text{ nos}}$