Bergen County Mathematics League

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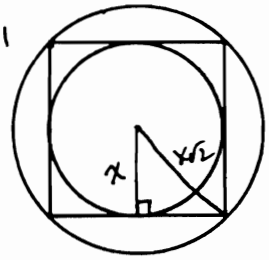
Brief Contest Solutions #6

2007-2008

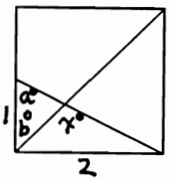
6-1



$\frac{\text{Area circumcircle}}{\text{Area incircle}} = \left(\frac{\sqrt{2}}{1}\right)^2 = \frac{2}{1}$, so area of the circumcircle $= 2(60) = \boxed{120}$.

6-2) Avg digit $= 3$. Avg $\# = 33333$. Since there are $5! = 120$ different such 5-digit numbers, their sum is $\boxed{120(33333)}$ or $\boxed{3999960}$

6-3)

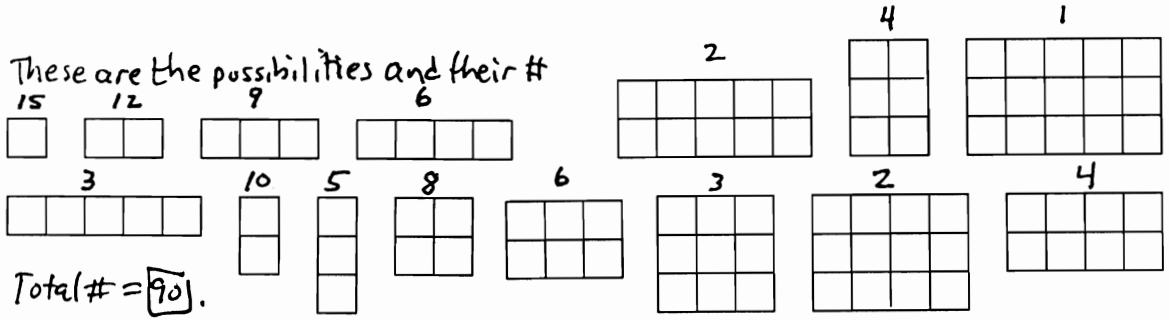


$\tan a^\circ = 2$
$\tan b^\circ = 1$

Since $x^\circ = a^\circ + b^\circ$

$\tan x^\circ = \frac{\tan(a^\circ) + \tan(b^\circ)}{1 - \tan(a^\circ)\tan(b^\circ)} = \frac{2+1}{1-2} = \boxed{-3}$.

6-4) These are the possibilities and their #



Total # $= \boxed{90}$.

6-5) Since $3\times4\times5 = 60$ and $3\times2\times2 = 12$, at most 5 of the smaller blocks can fit into the space. Since the $3\times4\times5$ space has four odd dimensions, and $3\times2\times2$ has two even dimensions, some space cannot be used. It is easy to see how to fit $\boxed{4}$ blocks in the space.

6-6) Since $\frac{1}{\sqrt{n}+\sqrt{n+1}} \cdot \frac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n}-\sqrt{n+1}} = \frac{\sqrt{n}-\sqrt{n+1}}{-1} = \sqrt{n+1}-\sqrt{n}$

Thus, $\sqrt{5}-\sqrt{4}+\sqrt{6}-\sqrt{5}+\ldots+\sqrt{n-2}-\sqrt{n-1}+\sqrt{n+1}-\sqrt{n}+\sqrt{n+1}-\sqrt{n} = 11$.

So, $\sqrt{n+1}-2 = 11$, $\sqrt{n+1} = 13$, so $n = \boxed{168}$.