Bergen County Mathematics League

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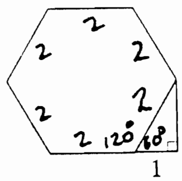
Brief Contest Solutions #1 **2009-2010**

1-1)



The interior angles of the regular hexagon are all 120° angles; so each side of the hexagon is twice the shorter leg of the 30°-60°-90° Δ, so each side is 2. The perimeter is $6 \times 2 = \boxed{12}$.

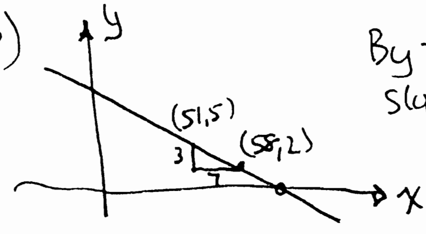
1-2) Since 4, 5, 6, 7, and 8 all divide $x-3$, $2^3 \cdot 7 \cdot 3 \cdot 5$ divides $x-3$ and is the smallest such divisor. Therefore 840 divides $x-3$, so $x = 840k+3$, $k \in Z^+$. The least such x is $\boxed{843}$.

1-3) The right side is the factorization of g^3-b^3, where $g = 3x$ and $b = 2\sqrt{2}$. Thus $a = (2\sqrt{2})^2 = 8$.
More easily, $-16\sqrt{2} = (-2\sqrt{2})(a)$, so $a = \boxed{8}$ by observation.

1-4) The prime factors of 10 are $2 \cdot 5$. The product $25!$ has many more factors of 2 than of 5, so each factor of 5 can produce one factor of 10 and one consequent 0. There is a 5 every 5th number. But there's an extra 5 in 25, so the number of factors of $5 = 5+1 = \boxed{6}$ = number of terminal 0s.

1-5) $(16^{\sqrt{2}})^x$ cannot represent 1 since $x > 0$. It can equal 2 if $(16^{\sqrt{2}})^x = (2^{4\sqrt{2}})^x = 2^{4x\sqrt{2}} = 2$. That happens if $x = \boxed{\frac{1}{4\sqrt{2}}}$.

1-6)



By trial, one solution is (58, 2). Using slope $= -\frac{3}{7}$, another solution is (51, 5). The general solution is $(58-7t, 2+3t)$. The solution with the least positive $y-x$ is $\boxed{(16,20)}$.