Bergen County Mathematics League

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BCML

Brief Contest Solutions #2

2009-2010

2-1) This is the Fibonacci sequence with different generators. Start with the first two terms (the generators) and add the last two to get the next. Continue forever to get

$$-1, 1, 0, 1, 1, 2, 3, 5, 8, \boxed{13}, 21, 34, 55, 89, \ldots$$

2-2) $d_1^2 + d_2^2 = 4^2 + 7^2 + 4^2 + 7^2 = 130$. Make a list of pairs of squares of integers that have a sum of 130

$9 + 121 = 130$ so lengths 3 and 11 ← not possible since $4+7=11$

$49 + 81 = 130$ 4 and $\boxed{9}$

2-3) A few examples should convince you that the product of the GCD and LCM of any two positive integers is equal to the product of these two integers. See a book on number theory for a proof. The product is $\boxed{xy}$.

2-4 The product $6 \cdot 7 \cdot 8 \cdot 9$ must divide $x-5$. The least such product is $7 \cdot 8 \cdot 9 = 504$. Thus, $x = 504k + 5$, $k \in \mathbb{Z}^+$ and the least such integer is $\boxed{509}$.

2-5) $x = \sqrt{2y}$, so $x^2 = 2y$. Since $y = \sqrt{x}$, $2y = 2\sqrt{x}$. Thus, $x^2 = 2\sqrt{x}$, so $x^4 = 4x$, so $x(x^3 - 4) = 0$. Since $x > 0$, $x = \sqrt[3]{4}$ and $y = \sqrt{x} = \sqrt{\sqrt[3]{4}} = \sqrt[3]{\sqrt{4}} = \sqrt[3]{2}$ and $(x, y) = \boxed{(\sqrt[3]{4}, \sqrt[3]{2})}$

2-6) Let m = "distance" moved by the minute hand. Then

$m/12$ = "distance" moved by the hour hand during same time period.

Thus, $m + 15 = \frac{m}{12} + 20$

Solving, $m = \boxed{5\frac{5}{11}}$.