Bergen County Mathematics League

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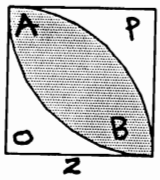
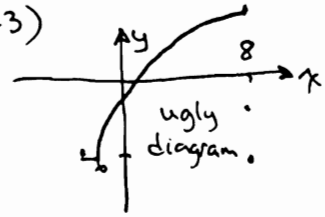
Brief Contest Solutions #4

2009-2010

4-1) For all b, $b>0, b\neq 1$, $\log_b b = \boxed{1}$.

4-2) 

The side of the square has length 2. The area of the quarter-circle AOB is $\frac{1}{4}\pi r^2 = \pi$. That's also the area of the quarter-circle centered at P, their sum is 2π. That include all of the square plus the overlap. The area of the square is 4, so the overlap's area is $\boxed{2\pi - 4}$.

4-3) 

The radius of the circle is 9.
The distance from $(8,-6)$ to the origin is 10.
There are no points in quadrant $\boxed{\text{II}}$

4-4) $2000^x = A$, $2000^y = B$, $2000^{2x} = A^2$, $2000^{2y} = B^2$

$A+B=9$, $AB=8$ $\}$ $\therefore (A,B) = (1,8)$ or $(8,1)$

$A^2+B^2 = (A+B)^2 - 2xy = 81-16 = \boxed{65}$.

(or $A^2+B^2 = 1^2+8^2 = 65$)

4-5)
$$\begin{array}{r} x+7 \\ x^2+1 \;\overline{)\; x^3+7x^2+Ax+B} \\ \underline{x^3 \qquad +x} \\ 7x^2+(A-1)x+B \\ \underline{7x^2 \qquad +7} \end{array}$$

Now, the remainder is $(A-1)x + (B-7)$. Thus, $(A,B) = \boxed{(1,7)}$.

4-6) Method I (can you explain this method?) $19+18+17+\ldots+3+2+1 = \frac{19}{2}(20) = \boxed{190}$.

Method 2 $\binom{20}{2} = 190$

Method III $\frac{20\times 19}{2} = 190$

Explanation of Method I: Man 20 shakes everyone else's hand, all 19, then leaves. Same for man 19, who shakes 18 other hands.