Bergen County Mathematics League

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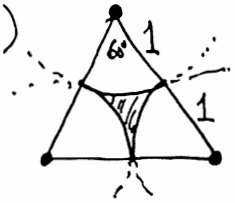
Brief Contest Solutions #5 — **2009-2010**

5-1) 

$r=1$, $s=2$

$\text{Area}_{\triangle} = \frac{s^2\sqrt{3}}{4} = \sqrt{3}$

$3\left(\frac{1}{6}\text{ circle}\right) = \frac{\text{circle}}{2} = \frac{\pi}{2}$

Shaded region = $\triangle$ − half-circle

$= \boxed{\sqrt{3} - \frac{\pi}{2}}$.

5-2) $269x + 231y = 288$

$231x + 269y = 212$

Add: $500x + 500y = 500$ or $x + y = 1$

Subtract: $38x - 38y = 76$ or $x - y = 2$

$x + y = 1$

$x - y = 2$

$2x = 3$

$x = \frac{3}{2}$

$(x,y) = \boxed{\left(\frac{3}{2}, -\frac{1}{2}\right)}$

5-3) $\text{Avg rate} = \frac{\text{Total distance}}{\text{total time}}$

one-way dist $= d$

$6 = \frac{2d}{\frac{d}{4} + \frac{d}{k}} = \frac{8dk}{d(k+4)} = \frac{8k}{k+4}$

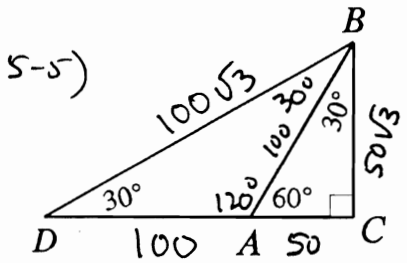
$\therefore 6k + 24 = 8k$

$k = \boxed{12}$

5-4) $(r+3)(10) = (3-r)(50)$

r = rate, in steps per second

$r = \boxed{2}$

5-5) 

$\boxed{50\sqrt{3}}$

5-6) $\log_b a = \frac{\log_c a}{\log_c b}$

if $a > 0$, $b > 0$, $b \neq 1$, $c > 0$, $c \neq 1$

$\Rightarrow (\log_{16} 9)(\log_3 25)(\log_5 4) = \frac{\log 9}{\log 16} \cdot \frac{\log 25}{\log 3} \cdot \frac{\log 4}{\log 5}$

$= \frac{2\log 3}{4\log 2} \cdot \frac{2\log 5}{\log 3} \cdot \frac{2\log 2}{\log 5}$

$= \boxed{2}$.

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