Bergen County Mathematics League

Good Luck To You — Good Luck To All

Contest #2 (Calculators Allowed) — **2010-2011**

Part I *Time Limit: 12 minutes* Answers must be exact ***or*** have 4 (or more) significant digits, correctly rounded.

2-1. How many integers greater than 99 and less than 1000 consist of only even digits?

2-2. A function f, defined on the positive integers, satisfies $f(n) = 3f(n-1) + 1$. If $f(1) = 0$, what is the value of $f(4003) - f(4001)$?

Part II *Time Limit: 12 minutes* Answers must be exact ***or*** have 4 (or more) significant digits, correctly rounded.

2-3. A 3×4 paper rectangle can be wrapped once around cylindrical pipe A so that two of its opposite edges just meet, or it can be wrapped once around cylindrical pipe B so that its other two opposite edges just meet. What is the ratio, smaller to larger, of the volumes of the portions of the pipes around which the paper rectangle can be wrapped?

2-4. If x, y, and z are non-zero numbers, what is the simplest form expression for the quotient $[x \div (y \div z)] \div [(z \div x) \div y]$?

Part III *Time Limit: 12 minutes*

2-5. What are all real values of x which satisfy $\dfrac{x^3 - x^2 - x + 1}{x^3 - x^2 + x - 1} = 0$?

2-6. Write $\dfrac{2 - \sqrt{8} + \sqrt{12}}{1 + \sqrt{2} - \sqrt{3}}$ in simplest radical form, NOT as a decimal.

***Notice*: A question on the next meet will repeat the theme of question 2-5.**

Answers

2-1. 100

2-2. $4(3^{4000})$ or $3^{4000} + 3^{4001}$

2-3. 3:4 or 3 to 4 or $\frac{3}{4}$ or 0.75, etc.

2-4. x^2

2-5. -1

2-6. $\sqrt{2} + \sqrt{6}$