Bergen County Mathematics League

Good Luck To You — Good Luck To All

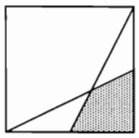
Contest #5 (No Calculators) — **2010-2011**

Part I *Time Limit: 12 minutes*

On contest #6, ***any S.A.T. calculator will be allowed.***

5-1. If $0 \leq x < 2\pi$, what are all values of x for which $\sin x + \cos x = 1$?

5-2. Line segments are drawn from two opposite vertices of a square to the midpoints of two sides that share a common vertex in the manner shown in the diagram at the right. If the area of the square is 900, what is the area of the shaded region?



Part II *Time Limit: 12 minutes*

5-3. What ordered pair of real numbers (x,y) satisfies $10^{\log(x+y)} = 50$ and $10^{\log(x-y)} = 48$, if both logarithms are base 10 logarithms?

5-4. What is the length of the common external tangent to two externally tangent circles whose radius-lengths are 4 and 9?

Part III *Time Limit: 12 minutes*

5-5. If $a \geq b \geq c > 0$, what is the square root of $(a^2+ab+ac+bc)(b^2+ab+ac+bc)(c^2+ab+ac+bc)$?

5-6. In $\triangle ABC$, $m\angle B = 30$, $AB = 8$, and $BC = 12$. Squares are drawn on $\overline{AB}$ and $\overline{BC}$ as sides of the squares so that the interiors of the squares have no points in common with the interior of $\triangle ABC$. If the centers of the squares are X and Y, what is the area of a square with side $\overline{XY}$?

***Notice*: Questions on the next meet will repeat the themes of questions 5-2 and 5-6.**

Answers

5-1. $0, \frac{\pi}{2}$

5-2. 150

5-3. (49,1)

5-4. 12

5-5. $(a+b)(a+c)(b+c)$ order of factors is irrelevant

5-6. 152