Part I *Time Limit: 12 minutes*   Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

6-1.   In how many points do the graphs of the polar curves $r = \sqrt{2}\sin\theta$ and $r = \sqrt{2}\cos\theta$ intersect when drawn in the xy-plane? (Note: Polar graphs are usually drawn in the xy-plane).

6-2.   For fixed real numbers $a, b,$ and c, and all real numbers x, let $f(x) = ax^5 + bx^3 + cx + 5$ define the function f. If $f(3) = 9$, what is the value of $f(-3)$?


Part II *Time Limit: 12 minutes*   Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

6-3.   If $0 \le x < 2\pi$, what is the least value of x for which $\sin x \cos x = \frac{1}{4}$?

6-4.   If lines ℓ and m are perpendicular to each other, what is the length of the graph of the locus of points the sum of whose distances from ℓ and m is 12?

Part III *Time Limit: 12 minutes*   Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

6-5.   Line segments are drawn from two opposite vertices of a square to the midpoints of two sides that share a common vertex in the manner shown in the diagram at the right. If the area of the square is 900, what is the area of the shaded region?
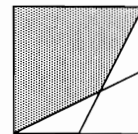
6-6.   The lengths of the legs of a certain right triangle are $6\sqrt{3}$ and $8\sqrt{3}$. A line segment joins the centers of the two equilateral triangles drawn in the exterior of the right triangle with the legs of the right triangle as sides of the equilateral triangles. What is the area of a square which has this line segment as one of its sides?

Answers

6-1.   2

6-2.   1

6-3. $\frac{\pi}{12}$ or $0.261799387799\ldots$

6-4. $48\sqrt{2}$ or $67.8822509939\ldots$

6-5.   600

6-6. $100 + 48\sqrt{3}$ or $183.138438763\ldots$