Bergen County Mathematics League

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Brief Contest Solutions #1

2010-2011

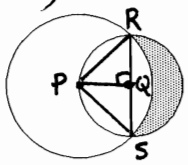
1-1) Combining $rc = 600$ and $(r-4)(c+5) = 600$, we get $r = \frac{600}{c}$ and $r = \frac{20+4c}{5}$. Equating $\frac{20+4c}{5} = \frac{600}{c}$, we get $4c^2 + 20c = 3000$, or $c^2 + 5c - 750 = (c-25)(c+30) = 0$, so $c = \boxed{25}$ and $r = 24$.

1-2)
$0.\overline{1} = 0.111111\ldots$
$0.\overline{12} = 0.121212\ldots$
$0.\overline{123} = 0.123123\ldots$
Sum = $\boxed{0.\overline{355446}}$

Since l.c.m$(1,2,3) = 6$, the sum will repeat with at most 6 digits.

1-3) Removing 20 nickels, the value of the remaining coins is \$100. Now, for every quarter, there are 5 dimes and 5 nickels. Since $1(25¢) + 5(10¢) + 5(5¢) = \$1.00$, the pile must have 100 quarters, 500 dimes, and $\boxed{520}$ nickels.

1-4)



Let P and Q be the respective centers of the larger and smaller circles. Since they are all radii of the smaller circle, $QS = QR = QP = 1$. Similarly, $PR = PS = \sqrt{2}$, so ΔQRP and ΔQSP are isosceles right triangles. The area of the lune is is half the area of $\odot Q$ minus the area of the smaller segment of $\odot P$, whose area is 1/4 the area of $\odot P$ minus the area of ΔPRS. The answer is $\frac{2\pi}{4} - (\frac{\pi}{2} - 1) = \boxed{1}$.

1-5) An integer is a multiple of 9 if and only if the sum of its digits is a multiple of 9. A number whose digit-sum is odd must have at least one odd digit, so the least sum of digits for our number is 18. Since 99 is not correct, the number must have 3 digits. Try a number in the 200's. For a sum of 18, with a hundred's digit of 2, the only possibility is $\boxed{288}$.

1-6) $\frac{12}{\sqrt{5}+(\sqrt{3}+\sqrt{2})} \cdot \frac{\sqrt{5}-(\sqrt{3}+\sqrt{2})}{\sqrt{5}-(\sqrt{3}+\sqrt{2})} = \frac{12(\sqrt{5}-\sqrt{3}-\sqrt{2})}{5-(\sqrt{3}+\sqrt{2})^2} = \frac{12(\sqrt{5}-\sqrt{3}-\sqrt{2})}{-2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} =$

$\frac{12(\sqrt{30}-\sqrt{18}-\sqrt{12})}{-12} = \sqrt{18}+\sqrt{12}-\sqrt{30} = \boxed{3\sqrt{2}+2\sqrt{3}-\sqrt{30}}$.