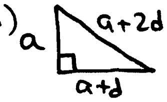
Bergen County Mathematics League

Problem Author:
Steve Conrad
www.mathleague.com

BCML

Problem Editor:
Dan Flegler
www.mathleague.com

Brief Contest Solutions #3 **2010-2011**

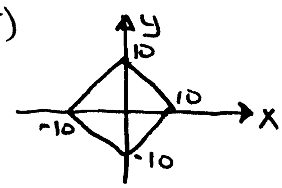
3-1) 

$a^2+(a+d)^2=(a+2d)^2 \Leftrightarrow a^2-2ad-3b^2=(a-3d)(a+d)=0$. Thus $a=3d$, $a+d=4d$, $a+2d=5d$. Since $5d=100$, $4d-3d=$ $d=\boxed{20}$.

3-2) Add 1 to both sides and rewrite the left side as a single fraction in simplest form to get $\frac{2x}{x-3}\ge 0$. This is true whenever both the numerator and denominator are positive or both are negative. This occurs for $\boxed{x>3 \text{ or } x<0}$.

3-3) $x+1=\dfrac{1}{1+\frac{1}{1+x}} \Leftrightarrow x+1=\dfrac{1}{\frac{x+2}{x+1}} \Leftrightarrow x+1=\dfrac{x+1}{x+2}$, $x\ne -1$. Solving $x+2=1$, we get $x=-1$, which is not a valid solution. There are $\boxed{\text{no real values}}$ of x that satisfy this equation.

3-4) Geometric: a, ar, ar^2
Arithmetic: $a+4, ar, ar^2 \Rightarrow ar^2-ar=ar-(a+4)$, or $a(r^2-2r+1)=-4$.
Thus, $a=\frac{-4}{(r-1)^2}$. Since a and r are both integers, $(r-1)^2$ is a divisor of -4. Only when $r=2,3,$ or -1 are viable solutions produced, and the solutions are $\boxed{(-1,-3,-9),(-4,-8,-16),(-1,1,-1)}$.

3-5) 

Area $=\frac{1}{2}(20\times 20)=\boxed{200}$.

3-6) The average of 1, 2, 3, 4, 5 is $\frac{1+2+3+4+5}{5}=3$, so the average such 4-digit number is 3333, and the sum of all 120 such numbers is $120\times 3333=\boxed{399960}$.