# Bergen County Mathematics League

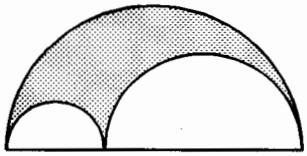
Good Luck To You — Good Luck To All

**Contest #4 (Calculators Allowed)** — **2011-2012**

**Part I** *Time Limit: 12 minutes* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

4-1. If $x^2 + 3xy = 54$ and $xy+4y^2 = 115$, what is the greatest possible value of $x+2y$?

4-2. Two semicircles with collinear diameters are externally tangent to each other at a trisection point of a larger semicircle to which each is internally tangent, as shown. If the area (in units$^2$) of the shaded region bounded by the three semicircles equals the perimeter (in units) of the same region, how long is the diameter of the largest of the three semicircles?

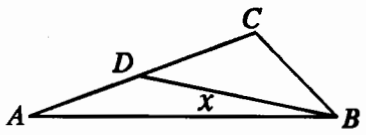


---

**Part II** *Time Limit: 12 minutes*

4-3. If function $f$, defined for all real numbers, satisfies $f(x) = 1-f(x-1)$, write an equation expressing $f(x+1)$ explicitly in terms of $f(x-1)$.

4-4. In $\triangle ABC$, $D$ is the point on $\overline{AC}$ for which $CD = CB$. If $m\angle ABC - m\angle CAB = 30$, what is the degree-measure of $\angle ABD$ (labeled $x$ in the diagram).




---

**Part III** *Time Limit: 12 minutes*

4-5. In how many different ways may a team win a series of up to 7 games if the first team to win 4 games wins the series? [**NOTE:** There are no tied games, and the series ends as soon as one team wins 4 games.]

4-6. What are all values of $x$ for which $\dfrac{1}{1+\dfrac{1}{1+\dfrac{x}{x-4}}}$ is undefined?

***Notice*: A question next meet will repeat the theme of question 2-3.**

---

**Answers**

4-1. 13

4-2. 18

4-3. $f(x+1) = f(x-1)$. The answer must be an EQUATION

4-4. 15 or 15°

4-5. 35 or $\binom{7}{4}$ or $\binom{7}{3}$

4-6. 2, $\frac{8}{3}$ 4