Bergen County Mathematics League

Good Luck To You — Good Luck To All

Contest #6 (Calculators Allowed) — **2011-2012**

Part I *Time Limit: 12 minutes* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

6-1. What is the value of the ratio of the least common multiple of 210 and 396 to their greatest common divisor?

6-2. The length R of a radius of circle I is 3 more than the length r of a radius of circle II, and the area of circle I is 3 times the area of circle II. For what ordered pair of rational numbers (a,b) does $r = a(1+\sqrt{b})$?

Part II *Time Limit: 12 minutes*

6-3. What are all ordered pairs of real numbers (x,y) which satisfy the simultaneous system

$$x^2 + 2xy - y^2 = 4,$$
$$x^2 - 3xy + y^2 = -4?$$

6-4. In quadrilateral $ABCD$, $\sin^2 A + \sin^2 B + \sin^2 C + \sin^2 D = 4$. If the area of $ABCD$ is 100 and each side of $ABCD$ has an integer length, what is the greatest possible perimeter of $ABCD$?

Part III *Time Limit: 12 minutes*

6-5. If a, b, c, and d are solutions of $x^4 - x^3 + x^2 - x + 1 = 0$, what is the numerical value of

$$a^5 + b^5 + c^5 + d^5 + 5a^5b^5c^5d^5?$$

6-6. A box contains 5 gold, 10 silver, and 15 copper coins. If 6 coins are chosen at random, without replacement, what is the probability that 1 of the 6 chosen coins is gold, 2 are silver, and 3 are copper?

Answers

6-1. 2310

6-2. $\left(\frac{3}{2},3\right)$

6-3. (2,4), (−2,−4)

6-4. 202

6-5. 1

6-6. $\frac{5}{29} = 0.172413793103\ldots$