Bergen County Mathematics League

Problem Author:
Steve Conrad
www.mathleague.com

Problem Editor:
Dan Flegler
www.mathleague.com

Brief Contest Solutions #3 — **2011-2012**

1) $2\left[\frac{16\pi}{2} - \frac{4\pi}{2}\right] = \boxed{12\pi}$

2) Method I: By observation, $(a,b,c) = (3,4,5)$. Finally, $4^2+5^2+6^2 = 16+25+36 = \boxed{77}$.

Method II

$2a+2b+2c = 24$

$a^2+2a+b^2+2b+c^2+2c = 50+24 = 74$

$(a+1)^2+(b+1)^2+(c+1)^2 = 74+3 = 77$

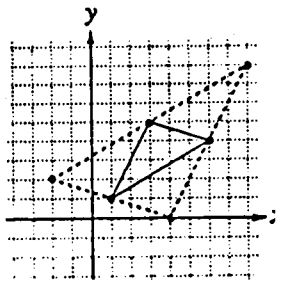
3) $\binom{m}{3} = \frac{m}{3} \Leftrightarrow \frac{m(m-1)(m-2)}{3\cdot 2\cdot 1} = \frac{m}{3} \Leftrightarrow (m-1)(m-2) = 2 \Leftrightarrow m^2-3m = 0$

$[m>0]$ ($m>0$) ($m>0$)

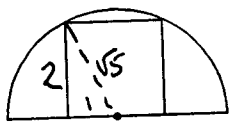
Thus, $m = \boxed{3}$.

4) There are 3 possibilities: (1,1) and (3,5) could be endpts of a diagonal; (1,1) and (6,4) could be endpoints of a diagonal; or (1,1) and (x,y) could be endpoints of a diagonal. Either $(1+3, 1+5) = (6+x, 4+y)$ or $(4+6, 1+1) = (3+x, 5+y)$ or $(1+x, 1+y) = (3+6, 5+4)$ since diagonals of a parallelogram have same midpt. Finally,

$(x,y) = \boxed{(-2,2), (4,0), (8,8)}$

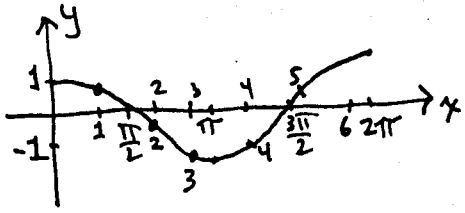


5)



The semicircles area is $\frac{\pi(\sqrt{5})^2}{2} = \boxed{\frac{5\pi}{2}}$.

6)



From the graph $\cos 3 < \cos 4 < \cos 2 < \cos 5 < \cos 1$

least is $\cos \boxed{3}$.