Bergen County Mathematics League

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Brief Contest Solutions #4

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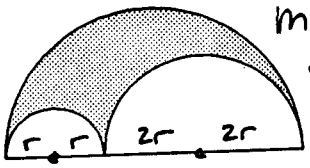
1) Adding the equations, $x^2+4xy+4y^2 = (x+2y)^2 = 169 = 13^2$, thus $x+2y = \pm 13$, and the largest possible value of $x+2y$ is $\boxed{13}$.

2) large semicircle's area = $\frac{\pi(3r)^2}{2}$

middle = $\frac{\pi(2r)^2}{2}$

small = $\frac{\pi r^2}{2}$



Shaded area = $\frac{\pi}{2}(9r^2-4r^2-r^2)$

Shaded perimeter = $\pi(3r+2r+r)$

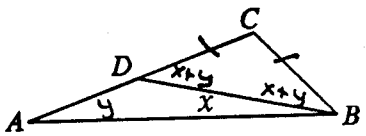
Thus, $2\pi r^2 = 6\pi r$, so $r=3$ and diam $= 6r = \boxed{18}$.

3) Since $f(x) = 1 - f(x-1)$, $f(x+1) = 1 - f(x)$. Therefore,

$f(x+1) = 1 - [1-f(x-1)] = f(x-1)$

$\boxed{f(x+1) = f(x-1)}$

4)



$m\angle ABC - m\angle CAB$

$= 2x+y - y$

$= 2x = 30$, so $x = \boxed{15}$.

5) Seven games are to be scheduled. I have to choose 4 of those games to win. The number of ways is $\binom{7}{4} = \binom{7}{3} = \boxed{35}$.

6) The fraction is undefined if any denominator is 0. For this fraction, there are 3 ways a denominator can be 0:

if $x-4=0$, $x=4$ | if $\frac{x}{x-4} = -1$, $x=2$ | if $1+\frac{x}{x-4} = -1$, $x = 8/3$

$\boxed{2, \frac{8}{3}, 4}$

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