Bergen County Mathematics League

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Brief Contest Solutions #5

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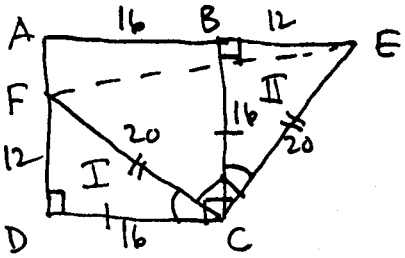
1) $a \parallel b \Rightarrow m_a = m_b$
$a \perp f \Rightarrow m_a = -\frac{1}{m_f}$
$e \parallel f \Rightarrow m_e = m_f = -\frac{1}{m_a} = -\frac{1}{m_b} = \boxed{\frac{-3}{2}}$.

Notice that line c plays no role

2) A total of 192 digits are used: 11 zeroes, 21 ones, 20 each twos, threes, fours, fives, sixes, sevens, eights, and nines. Their sum is 901. The mean is $\boxed{\frac{901}{192}}$.

3) 

$\Delta I \cong \Delta II$ by ASA
So $FC = EC = 20$ and ΔECF is isos rt Δ.
The shorter leg of ΔI & ΔII is $\boxed{12}$.

4) Since $A+B+C = 180°$, $\sin C = \sin(180° - (A+B)) = \sin(A+B)$
$\sin C = \sin A \cos B + \cos A \sin B = \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13} = \boxed{\frac{63}{65}}$.

5) $0.\overline{24} = \frac{24}{99} = \frac{8}{33}$. Let $y = 0.1\overline{33}...$, so $10y = 1.3\overline{33}...$ and $9y = 1.2$,
Thus, $y = \frac{12}{90} = \frac{2}{15}$.
Finally $\frac{8}{33} \times \frac{15}{2} = \frac{20}{11}$, so $x = \boxed{20}$.

6) $(x+1)(x^4 - x^3 + x^2 - x^1 + 1) = (0)(x+1) \Leftrightarrow x^5 + 1 = 0$. Thus, for every root r of both this and original equation, $r^5 = -1$. Thus,
$x^{40} - x^{30} + x^{20} - x^{10} + 1 = (-1)^8 - (-1)^6 + (-1)^4 - (-1)^2 + 1 = 1 - 1 + 1 - 1 + 1 = \boxed{1}$.