Bergen County Mathematics League

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Brief Contest Solutions #6 **2011-2012**

1) $\text{lcm}(210, 396) = 2^2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$ $\quad 210 = 2\cdot3\cdot5\cdot7 \quad 396 = 2^2\cdot3^2\cdot11$
$\text{gcd}(210, 396) = 2\cdot 3$
$$\frac{\text{lcm}}{\text{gcd}} = \frac{2^2\cdot3^2\cdot5\cdot7\cdot11}{2\cdot3} = 2\cdot3\cdot5\cdot7\cdot11 = 3\times10\times77 = 3\times770 = \boxed{2310}.$$

2) $R = r+3$; $\pi R^2 = 3\pi r^2$, so $\cancel{\pi}(r+3)^2 = 3\cancel{\pi}r^2 \Rightarrow \left(\frac{r+3}{r}\right)^2 = \left(1+\frac{3}{r}\right)^2 = 3$,
so $1+\frac{3}{r} = \sqrt{3} \Rightarrow \frac{3}{r} = \sqrt{3}-1 \Rightarrow r = \frac{3}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{3\sqrt{3}+3}{2} = \frac{3}{2}(1+\sqrt{3})$, so
$$(a,b) = \boxed{\left(\tfrac{3}{2}, 3\right)}$$

3)
$$x^2+2xy-y^2 = 4$$
$$x^2-3xy+y^2 = -4$$
Adding $2x^2 - xy = 0 \Rightarrow x = 0$ or $y = 2x$.

If $x=0$, y is imaginary. By substituting $y=2x$ into either of the original equations, $(x,y) = \boxed{(2,4),(-2,-4)}$

4) Each $\measuredangle$ is a right $\measuredangle$, so the quadrilateral is a rectangle. When 2 sides are 1 each and the other two are 100 each, the area is 100 and the perimeter is maximized, for integer-length sides, at $100+100+1+1 = \boxed{202}$.

5) As shown in the solution to 5-6 (last contest), $r^5 = -1$ for each root r. Thus, $a^5+b^5+c^5+d^5+5a^5b^5c^5d^5 = (-1)^5+(-1)^5+(-1)^5+(-1)^5+5(-1)^5(-1)^5(-1)^5(-1)^5$
$= -4+5 = \boxed{1}$.

6) $$P(1\text{ gold}, 2\text{ silver}, 3\text{ copper}) = \frac{\binom{5}{1}\binom{10}{2}\binom{15}{3}}{\binom{30}{6}} = \frac{5\times\frac{10\times9}{2}\times\frac{15\times14\times13}{3\times2}}{\frac{30\times29\times28\times27\times26\times25}{6\times5\times4\times3\times2}}$$
$$= \boxed{\frac{5}{29}}.$$