Bergen County Mathematics League

Good Luck To You

Good Luck To All

Contest #6 (Calculators Allowed) **2012-2013**

Part I *Time Limit: 12 minutes* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

6-1. Let any 3 consecutive terms of the arithmetic sequence 1, 4, 7, 10, . . . be called a, b, and c respectively. What are the rectangular coordinates of the point common to the graphs of all equations of the form $ax + by = c$?

6-2. Factor completely into 3 linear polynomials with integral coefficients:

$$a(b - c)^2 + b(c - a)^2 + c(a - b)^2 + 8abc.$$

Part II *Time Limit: 12 minutes*

6-3. Alternate trisection points of the sides of equilateral $\triangle ABC$ are vertices of equilateral $\triangle DEF$. What is the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$?

6-4. If $\log_y x + \log_x y = \frac{10}{3}$, then y has two possible values in terms of x. Write two DIFFERENT EQUATIONS, each expressing y explicitly in terms of x.

Part III *Time Limit: 12 minutes*

6-5. In 2 hours, A can run 1 km further than B. With a 2 minute headstart, B could tie A in a 4 km race. If all speeds are constant, what is B's rate (in km per hour)?

6-6. If $f(x) = \sqrt{\frac{x-1}{x+1}}$ and $g(x) = \frac{\sqrt{x-1}}{\sqrt{x+1}}$, what is the largest integer x in the domain of f that's **NOT** in the domain of g?

Answers

6-1. $(-1,2)$

6-2. $(a + b)(a + c)(b + c)$

6-3. 3

6-4. $y = x^3$, $y = x^{1/3}$

6-5. $7\frac{1}{2}$ or $7\frac{1}{2}$ km/h or exact equivalent

6-6. -2