Bergen County Mathematics League

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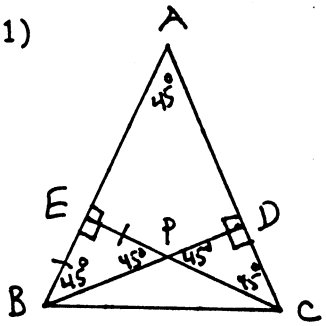
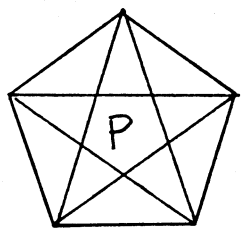
Brief Contest Solutions #2 — **2012-2013**

2-1)



2-2) Non-overlapping and outside P : 10
Just pairs of △ " " " : 10
Made up of 3△ " " " : 5
Containing P, with 2 vertices (only) of original pentagon : 5
Containing P, with 3 vertices of original pentagon : 5
35



2-3) $mb = (m+2)(b-100) = (m+5)(b-200)$

$mb = mb + 2b - 100m - 200$; $mb = mb + 5b - 200m - 1000$

$b - 50m = 100$
$b - 40m = 200$ $\Rightarrow$ $m = 10$ and $b = \boxed{600}$

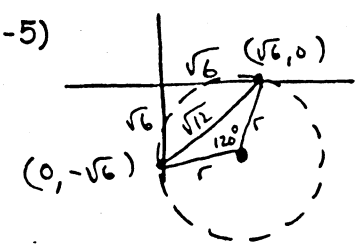
2-4) $-(x^3 + px^2 - 3x + 4 = 0)$

$x^3 + px^2 - 5x + 8 = 0$

$-2x + 4 = 0 \Rightarrow x = 2$ is a root.

$\therefore 8 + 4p - 10 + 8 = 0 \Rightarrow p = \boxed{-\frac{3}{2}}$

2-5)



$(\sqrt{12})^2 = r^2 + r^2 - 2r^2\cos 120°$

Thus, $\boxed{r = 2}$

2-6) For $a, b > 0$ $\quad a^b = b^a \Leftrightarrow a^{1/a} > b^{1/b}$

Since $x^{1/x}$ is strictly DEcreasing for $x > e$ and tends to 1 as $x \to \infty$, it follows that $3^{1/3} > 4^{1/4} > 5^{1/5} > \ldots$

So the only positive solutions are $\boxed{(2,4), \quad (4,2)}$

The other solutions have $a, b < 0$. $\boxed{(-2,-4), (-4,-2)}$