Bergen County Mathematics League

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Brief Contest Solutions #3

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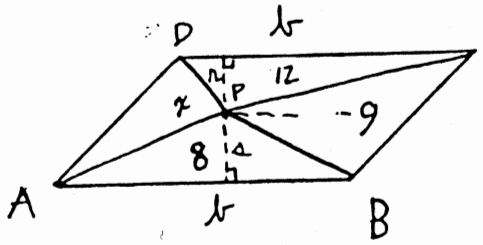
3-1) $\left(x^3+3x+\frac{3}{x}+\frac{1}{x^3}\right)+\left(x^3-3x+\frac{3}{x}-\frac{1}{x^3}\right)=\frac{6}{x}+16$

$\therefore 2x^3=16$ and $x=\boxed{2}$.

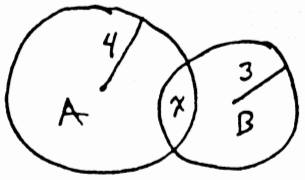
3-2) 

$K_{APB}=\frac{bh}{2}$, $K_{CDP}=\frac{br}{2}$

$K_{APB}+K_{CDP}=\frac{b}{2}(r+s)=\frac{1}{2}K_{ABCD}$

Similarly, $K_{APD}+K_{BPC}=\frac{1}{2}K_{ABCD}$

$\therefore x=(8+12)-9=\boxed{11}$.

3-3) 

$A+x=16\pi$
$B+x=9\pi$
$A-B=\boxed{7\pi}$.

3-4) $\left(\sqrt[6]{x}-2\right)\left(\sqrt[6]{x}+2\right)-\left(x+3\sqrt[3]{x^2}+3\sqrt[3]{x}+1\right)^{1/3}$

$=\left(\sqrt[3]{x}-4\right)\cdot\left(\left(x^{1/3}+1\right)^3\right)^{\frac{1}{3}}=\cancel{\sqrt[3]{x}}-4-\cancel{\sqrt[3]{x}}-1=\boxed{-5}$ for all x.

3-5) $36\left(1\frac{1}{3}-1\frac{2}{7}\right)=\cancel{36}\left(\frac{1}{\cancel{21}}\right)=\frac{12}{7}$ or $\boxed{1\frac{5}{7}}$.

3-6) Squaring and combining like terms,

$4x^2+2\sqrt{(2x^2+2)^2-(3x)^2}+4=7x^2+8$, so

$2\sqrt{(2x^2+2)^2-(3x)^2}=3x^2+4$.

Squaring again, $7x^4-28x^2=0$, so $x=\boxed{0,2,-2}$.