Bergen County Mathematics League

Problem Author:
Steve Conrad
www.mathleague.com

Brief Contest Solutions #4

Problem Editor:
Dan Flegler
www.mathleague.com

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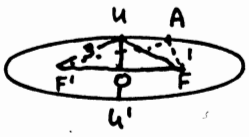
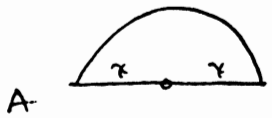
4-1) $\left.\begin{array}{l} a-b=2 \\ a+b=2x \end{array}\right\} \Rightarrow a=1+x$ and $b=-1+x$

$ax+by=(1+x)x+(-1+x)y=x+x^2-y+xy=x^2+xy$

$\therefore x-y=0$ and $\boxed{y=x}$

4-2) 

$AF+AF'=4$, so $UF+UF'=4$
Since $UF=UF'$, $UF=2$. Since
$UO=FO$, ΔUFO is an isos rt. Δ.
with $OF=OF'=\sqrt{2}$. In $\Delta AFF'$, $AF=1$,
$FF'=2\sqrt{2}$, and $AF'=3$, so $m\angle AFF'=\boxed{90}$

4-3) 

$\pi x-2x=114$

$x=\frac{114}{\pi-2}=\frac{114}{1.14}=100$

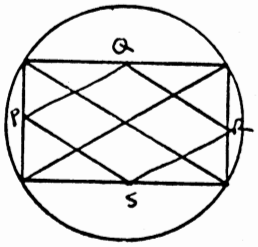
$2x=AB=\boxed{200}$

4-4)

	For	Against
men	x	$600-x$
women	$2x$	$600-2x$

$2x-(600-2x)=4[(600-x)-x]$

so $x=\boxed{250}$

4-5) 

PQRS is a rhombus, so diagonals of rectangle are each $2(3)=6$ and $r=\boxed{3}$

4-6) Let $F(x)=$ sum. Then for $x=a, b, c, 0$, $F(x)-x=0$.
Since $F(x)-x$ is a cubic polynomial that vanishes for 4 values of x, it is identically 0.

Thus, $F(x)=\boxed{x}$