**Bergen County Mathematics League**

Problem Author:
Steve Conrad
www.mathleague.com

Problem Editor:
Dan Flegler
www.mathleague.com

**Brief Contest Solutions #5**

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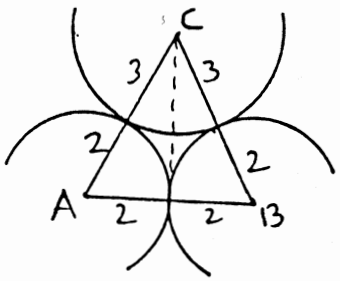
5-1) The diagonals intersect at $(0,-12)$. Hence, the fourth vertex must be at $(-9,-24)$.

5-2) $3^{2x+2} - 6(3^x) + 1 = 9(3^{2x}) - 6(3^x) + 1 = \left[(3)(3^x) - 1\right]^2 = 0$

Thus, $3^{x+1} = 1$ and $x = \boxed{-1}$

5-3)




Alt. to $\overline{AB} = \sqrt{5^2 - 2^2} = \sqrt{21}$

Area $= \frac{1}{2}(2+2)\sqrt{21} = 2\sqrt{21} = \sqrt{84}$

$\therefore k = \boxed{84}$

5-4) $(\log_2 x)^2 - 2\log_2 x - 8 = 0$

$(\log_2 x - 4)(\log_2 x + 2) = 0$

$\therefore \log_2 x = 4, \quad \log_2 x = -2$

$x = 2^4 = 16$ or $x = 2^{-2} = \frac{1}{4} = \boxed{\left\{16, \frac{1}{4}\right\}}$

5-5) $\cos^4 x + \sin^4 x = 1 \Rightarrow (\cos^2 x + \sin^2 x)^2 = 1 + 2\sin^2 x \cos^2 x = 1$

$\Rightarrow 0 = \sin x \cos x$

So solution set is $\boxed{\left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right\}}$

5-6)

$$\frac{1}{1\cdot 2\cdot 3} = \frac{1}{2}\left[\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3}\right]$$

$$\frac{1}{2\cdot 3\cdot 4} = \frac{1}{2}\left[\frac{1}{2\cdot 3} - \frac{1}{3\cdot 4}\right]$$

$$\vdots \qquad \vdots$$

$$\frac{1}{8\cdot 9\cdot 10} = \frac{1}{2}\left[\frac{1}{8\cdot 9} - \frac{1}{9\cdot 10}\right]$$

Sum $= \frac{1}{2}\left[\frac{1}{1\cdot 2} - \frac{1}{9\cdot 10}\right] = \boxed{\frac{11}{45}}$