Bergen County Mathematics League

Problem Author:
Steve Conrad
www.mathleague.com

Problem Editor:
Dan Flegler
www.mathleague.com

Brief Contest Solutions #6

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6-1) $a_{n+1} - a_n = a_n - a_{n-1} = 3$

$\therefore (-1)\, a_{n-1} + 2a_n = a_{n+1}$

$\uparrow x \qquad \uparrow y$

All pass through $\boxed{(-1, 2)}$

6-2) Let the expression be $E(a,b,c)$.

Then $E(-b,b,c) = -b(b^2 - 2bc + c^2) + b(c^2 + 2bc + b^2) + c(4b^2) - 8b^2c = 0$

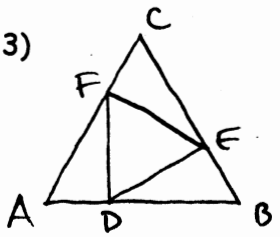
We could then expect $(a+b)$ to be a factor.

Similarly, we could expect $(b+c)$ and $(c+a)$ to be factors.

Hence, for some k, we expect $E(a,b,c) = k(a+b)(b+c)(c+a)$.

But, this equality is an identity. Equating coefficients of a^2b, $1 = k$. Thus $E(a,b,c) = \boxed{(a+b)(a+c)(b+c)}$.

6-3)



$K_{ADF} = \frac{2}{9} K_{ABC} = K_{BED} = K_{ECF}$

$K_{ADF} + K_{BED} + K_{ECF} = \frac{6}{9} K_{ABC}$

$\therefore K_{DEF} = \frac{3}{9} K_{ABC} = \frac{1}{3} K_{ABC}$ and $\boxed{k = 3}$

6-4) Let $u = \log_y x$. Then $\log_x y = \frac{1}{u}$ and $u + \frac{1}{u} = \frac{10}{3} = 3 + \frac{1}{3}$

If $u = 3$, $\boxed{y = x^{1/3}}$

If $u = \frac{1}{3}$, $\boxed{y = x^3}$

6-5) a = speed of A, b = " " B $\Rightarrow$ $2a = 2b + 1$ (A runs 1 km more)

$\frac{4}{a} = \frac{4 - \frac{b}{30}}{b}$ (Time for A = Time for B)

$\therefore 120b = 120a - ab$

$120b = 60(2b+1) - \frac{b}{2}(2b+1)$

$b^2 + \frac{b}{2} - 60 = 0$

$(2b - 15)(b + 8) = 0$

$b = \boxed{7\frac{1}{2}}$

6-6) The domain of f is $x < -1$ or $x \geq 1$

$g \qquad x \geq 1$

Domain of F - Domain of g is $x < -1$, so largest such integer is $\boxed{-2}$.