Bergen County Mathematics League

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Brief Contest Solutions #1

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1-1. $x - \frac{2}{3}x - \frac{3}{4}(x - \frac{2}{3}x) = 18 \Rightarrow x = \boxed{216}$

1-2. Since $A=5$, $C=2$. Then, $C=2 \Rightarrow E=1 \Rightarrow B$ is even.
Since $A=5$, $H \neq 5$. Then, $H=0 \Rightarrow D$ is even.
Since $5A + \text{carry} \geq 26$, $D \geq 6$, so $D = 6$ or 8.
If $D=6$, $B=3$. But B is even.
Thus, $D=8$, $B=6$, and $ABCCBD$ is $\boxed{562268}$.

1-3. $a^2 = \frac{4+2\sqrt{3}}{8}$, $b^2 = \frac{4-2\sqrt{3}}{8}$, $c^2 = \frac{3}{4}$, $2ab = \frac{4}{8}$.
$\therefore \frac{a^2+b^2-c^2}{2ab} = \frac{8-6}{8} \times \frac{8}{4} = \boxed{\frac{1}{2}}$.

Method II: $\frac{(a^2+2ab+b^2)-c^2-2ab}{2ab} = \frac{(a+b)^2-c^2}{2ab} - 1$
$= \frac{(1+\frac{1}{8})-(\frac{3}{4})}{(\frac{1}{2})} - 1 = \boxed{\frac{1}{2}}$

1-4. Avg rate $= \frac{\text{total distance}}{\text{total time}} = \frac{2d}{\frac{d}{20}+\frac{d}{30}} = \boxed{24}$.

1-5. orig: $A = \pi r^2$
new: $A = (\frac{4}{5}r)^2 \cdot \pi = \frac{16\pi r^2}{25}$, a $\boxed{36\%}$ decrease

1-6. $(x+1)^2 + y^2 = 4+1$
if $x+1 = \pm 2$, $y = \pm 1$
$x+1 = \pm 1$, $y = \pm 2$