Bergen County Mathematics League

Problem Author:
Steve Conrad
www.mathleague.com

Problem Editor:
Dan Flegler
www.mathleague.com

Brief Contest Solutions #6 **2013-2014**

6-1. $\left.\begin{matrix} A+C > B+D \\ A+B = C+D \end{matrix}\right\}$ Given

$\therefore C-B > B-C \Rightarrow \boxed{C>B}$

$A+B = C+D$

$A+\cancel{B+C} > \cancel{B+C}+D \Rightarrow \boxed{A>D}$

Also, $\boxed{D>C}$ is given.

Combining, $\boxed{A>D>C>B}$

6-2. $\log_B A = \frac{\log A}{\log B} = \frac{\cancel{2014}}{\cancel{2014}} \cdot \frac{\log 8}{\log 4} = \log_4 8 = \boxed{\frac{3}{2}}$

6-3. $xy = 2x+2y \Rightarrow x = \frac{2y}{y-2} = 2 + \frac{4}{y-2}$

Thus, $y = 4$ or 6 and $(x,y) = \boxed{(4,4),(3,6)}$

$x = 4$ or 3

6-4. $\sqrt{x}\left(\sqrt{7}-\sqrt{3}\right) = 7-3 \Rightarrow \sqrt{x} = \sqrt{7}+\sqrt{3}$

So $x = \left(\sqrt{7}+\sqrt{3}\right)^2 = \boxed{10+2\sqrt{21}}$

6-5. $\left.\begin{matrix} \text{Al: } B = S+1 \\ \text{Sister: } B = 2(S-1) \end{matrix}\right\} \Rightarrow (B,S) = \boxed{(4,3)}.$

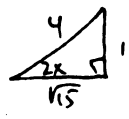
6-6. $\cos 2x = \cos^2 x - \sin^2 x \Leftrightarrow \sin^2 x - \cos^2 x = -\cos 2x$, or

$(\sin x - \cos x)(\sin x + \cos x) = -\cos 2x$

and $(\sin x - \cos x) = -\frac{\cos 2x}{\sqrt{5}/2}$

Also, $(\sin x + \cos x)^2 = \frac{5}{4}$

So $\sin^2 x + \cos^2 x + 2\sin x \cos x = 1 + \sin 2x = \frac{5}{4}$ and $\sin 2x = \frac{1}{4}$



so $\cos 2x = \frac{\sqrt{15}}{4}$

$\therefore \sin x - \cos x = -\cos 2x \cdot \frac{2}{\sqrt{5}} = \frac{-\sqrt{15}}{4} \cdot \frac{2}{\sqrt{5}} = \boxed{\frac{-\sqrt{3}}{2}}.$