Bergen County Mathematics League

Good Luck To You

Good Luck To All

Contest #4 (Calculators Allowed) **2014-2015**

Part I *Time Limit: 12 minutes* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

4-1. What is the area of the rectangle bounded by the lines $x = 0$, $y = 0$, $y = 9$, and the image of $y = 9$ under a counterclockwise quarter-turn about $(3,4)$?

4-2. Find, in simplest form, the ordered pair of real numbers (k,n) for which the expansion, by the binomial theorem, of $n = (1 + k)^k$, takes the form

$$n = 1 + \frac{1}{4} + \frac{1\times 3}{4\times 8} + \frac{1\times 3\times 5}{4\times 8\times 12} + \ldots.$$

Part II *Time Limit: 12 minutes*

4-3. Ty Kune always wears the red tie he got as a gift from Russian tea magnate Sam O. Var when the two met in the town of Chris, Mass. Ty owns two suits (blue and grey), two shirts (blue and white), and socks of two colors (blue and white). If Ty wears his blue suit and a white shirt, he also wears a blue tie. Ty always wears blue socks with his grey suit. Ty never wears his blue suit unless he also wears blue socks or a white shirt. Whenever Ty wears blue socks, he wears a white shirt. Besides the red tie, what color suit, shirt, and socks does Ty wear on his dates with his girlfriend, Miss Fortune?

4-4. In how many points do the graphs of $y = x^2$ and $y = \sin 8x$ intersect?

Part III *Time Limit: 12 minutes*

4-5. Five raffle tickets are drawn at random without replacement from a set of 149 tickets numbered consecutively from 1 to 149. What is the probability that the five ticket numbers were drawn in strictly increasing order?

4-6. Since $64 = 8^2 = 4^3$, we see that 64 is the smallest whole number greater than 1 that's simultaneously a square and a cube (*i.e.*, a number that can be written both as a second and a third power of an integer). What is the smallest whole number greater than 1 that is simultaneously a square, a cube, a 4th power, a 5th power, and a 6th power of an integer?

***Reminder*: A question next meet will repeat the theme of question 3-4.**

Answers

4-1. 18

4-2. $(-\frac{1}{2},\sqrt{2})$ or $(-0.5,\sqrt{2})$

4-3. grey suit, white shirt, blue socks

4-4. 6

4-5. 1/5! or 1/120 or $0.008\overline{3}$

4-6. 2^{60} or 1 152 921 504 606 846 976