Bergen County Mathematics League

Good Luck To You — Good Luck To All

Contest #6 (Calculators Allowed) — **2014-2015**

Part I *Time Limit: 12 minutes* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

6-1. If $\overline{X}$ is the average of the x-coordinates, and $\overline{Y}$ is the average of the y-coordinates, of all 12 integral solutions (x,y) of $x^2 + y^2 = 25$, what is the ordered pair $(\overline{X},\overline{Y})$?

6-2. What is the domain of the function f for which $f(x) = \sqrt{x^4 + 2x^3 - 8x^2}$?

Part II *Time Limit: 12 minutes*

6-3. It can be proven that $\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2$. If $\sum_{i=1}^{20} i^3 = k^2$ and $k > 0$, what is the value of k?

6-4. What are all real values of x for which $\frac{x^2-36}{x^2-9} = 1 - \frac{27}{x^2-9}$ is an integer?

Part III *Time Limit: 12 minutes*

6-5. I built a ceramic container with a capacity of 1 liter. My sister built a container in exactly the same shape and with exactly the same relative proportions as my container, but the linear dimensions of her container were twice those of mine. What is the capacity of her container, in liters?

6-6. Factor as far as possible into polynomials with integral coefficients:

$$a^2(b - c) - b^2(a - c) + c^2(a - b).$$

Answers

6-1. (0,0)

6-2. $x \leq -4$ or $x = 0$ or $x \geq 2$ (or exactly equivalent disjunction)

6-3. 210

6-4. $0, \pm\sqrt{6}, \pm\sqrt{8}, \pm\sqrt{10}, \pm\sqrt{12}, \pm\sqrt{18}, \pm 6$

6-5. 8 or 8 liters

6-6. $(a - b)(b - c)(a - c)$