Bergen County Mathematics League

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Brief Contest Solutions #1

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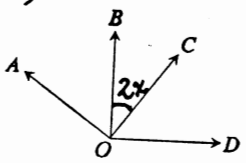
1-1) $10^2+11^2+12^2 = 100+121+144 = 365 \Rightarrow 13^2+x^2 = 169+x^2$, so $x^2 = 196$ & $x = \boxed{\pm 14}$.

1-2) By adding all 5 equations together, we get $6a+6b+6c+6d+6e = 28$. Dividing both sides by 6, we get $a+b+c+d+e = \frac{28}{6} = \boxed{\frac{14}{3}}$.

1-3)



Let $m\angle BOC = 2x$. Since $m\angle AOD = 4(m\angle BOC) = 4(2x) = 8x$, and since $\angle AOC \cong \angle BOD$, $m\angle AOB = m\angle COD = 3x$. Since $m\angle AOC = 90$, $5x = 90$, so $m\angle BOC = 2x = \boxed{36}$.

1-4) If x is a root of $9x^3-36x^2+44x-16 = 0$, then $\frac{1}{x}$ will be a root of $9(\frac{1}{x})^3 - 36(\frac{1}{x})^2 + 44(\frac{1}{x}) - 16 = 0$. $9-36x+44x^2-16x^3 = 0$ is the result after clearing fractions. The roots of $-16x^3+44x^2-36x+9 = 0$ are $\frac{1}{2/3}, \frac{1}{4/3}, \frac{1}{2}$, or $\boxed{\frac{3}{2}, \frac{3}{4}, \frac{1}{2}}$.

1-5) If $n=1$, $n^3+11n = 12$. If $n=2$, $n^3+11n = 30$. Clearly, the largest such divisor is ≤ 6. Here is a proof that the answer is 6: $n^3+11n = n^3-n+12n = n(n^2-1)+12n$ $= (n-1)(n)(n+1)+12n$. It can be proven that the product of any 3 consecutive integers is divisible by both 2 and 3, therefore by $\boxed{6}$.

1-6) The median of S is not in S, so S must have an even number of elements. Thus, half the numbers in S are less than the median, and half are greater. The probability that the 6 chosen numbers are all $>$ the median is $(\frac{1}{2})^6 = \frac{1}{64}$. Similarly the probability the 6 numbers are all $<$ the median $= (\frac{1}{2})^6$. We are looking for the complementary event, so $P = 1-\frac{1}{64}-\frac{1}{64} = \boxed{\frac{31}{32}}$.