Bergen County Mathematics League

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Brief Contest Solutions #3

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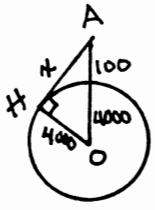
3-1) The given expression is the expression that results from expanding $(86-87)^5$ by the binomial theorem, so the value is $(-1)^5 = \boxed{-1}$.

3-2) Let $y = x^2+2x$, and then $\frac{1}{y-4} + \frac{18}{y+1} - \frac{18}{y} = 0$. Clearing fractions, we get $y^2-17y+72 = (y-9)(y-8) = 0$. If $y = x^2+2x = 9$, then $x = \boxed{-4, 2}$. If $y = x^2+2x = 8$, we get no other integral solutions.

3-3) $(x+y)^2 = x^2+y^2+2 \Leftrightarrow x^2+2xy+y^2 = x^2+y^2+2 \Leftrightarrow xy = 1$. The only integer solutions are $\boxed{(1,1), (-1,-1)}$.

3-4) It is clear that if $m>6$ AND $n>6$, there is no solution possible for $\frac{4}{m}+\frac{2}{n}=1$. There is a solution for (6,6). For any other solution, either m or n (not both) must be <6. Clearly $m = 1, 2, 3, 4$ do not work. For $m=5$, $\frac{4}{5}+\frac{2}{n}=1$ when $n=10$, so (5, 10) is a solution. Now try $n = 1, 2, 3, 4, 5$. The four solutions are $\boxed{(6,6), (5,10), (12,3), (8,4)}$.

3-5) 

$x^2 + 4000^2 = 4100^2$, so $x = \sqrt{4100^2-4000^2} = \sqrt{(4100+4000)(4100-4000)} = \sqrt{(8100)(100)} = 90 \times 10 = \boxed{900}$.

3-6)

	Now	Then	Future
Man	6x	5x	7x
Son	5x	4x	6x

Let $6x$ = man's current earnings, in dollars

$7x+6x = 117000$, so $x = 9000$

and $6x = 6(9000) = \boxed{\$54,000}$