Bergen County Mathematics League

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Brief Contest Solutions #5

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5-1) Let P' have coordinates (x,y). The point $(2,1)$ is the midpoint of $\overline{PP'}$. Thus, $\frac{x-\sqrt{2}}{2}=2$ and $\frac{y+\sqrt{2}}{2}=1$. Solving, $(x,y)=\boxed{(4+\sqrt{2},\ 2-\sqrt{2})}$.

5-2) The base must be a positive number other than 1, so $3-x>0$, $x\neq 2$. The argument must be positive, so $x+1>0$, or $x>-1$. Altogether, $\boxed{-1<x<3,\ x\neq 2}$.

5-3) Since $\frac{1}{18}+\frac{1}{18}=\frac{1}{9}$, it follows that $m>18>n>0$. Clearly $18>n>9$. By trial $(m,n)=\boxed{(90,10),(36,12)}$.

5-4) $\frac{x^2-9}{x^2-4}$ is an integer whenever x^2-4 is a factor of 5 since $\frac{x^2-9}{x^2-4}=1\frac{-5}{x^2-4}$. Solving $x^2-4=\pm 1$ or ± 5, $x=\boxed{\pm\sqrt{3},\ \pm\sqrt{5},\ \pm 3}$.

5-5) The circles are centered at $(0,0)$ and $(5,12)$ so the distance between their centers is 13. The lengths of the radii are 3 and 4, so the distance between the circles, but outside the circles, is $13-(3+4)=\boxed{6}$.

5-6) If the octagon were flattened out to a straight line, the answer would be 10. But, our circle must also turn through each exterior angle of the octagon, and the sum of the exterior angles is always 360°, thus requiring 1 additional revolution, for a total of $\boxed{11}$.