Bergen County Mathematics League

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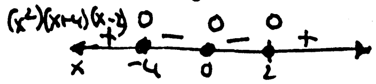
Brief Contest Solutions #6 **2014-2015**

6-1) For every pair of integers (x,y) that satisfies $x^2+y^2=25$, the pair $(-x,-y)$ is also a pair of integers that satisfy $x^2+y^2=25$. The average of all such pairs is $(\bar{x},\bar{y})=\boxed{(0,0)}$.

6-2) The requirement is that $x^4+2x^3-8x^2 = x^2(x+4)(x-2) \geq 0$



$\boxed{\{x \mid x \leq -4 \text{ or } x=0 \text{ or } x \geq 2\}}$ or equivalent.

6-3) $\sum_{i=1}^{20} i = 1+2+3+\ldots+20 = \frac{n}{2}(a_1+a_n) = 10(21) = 210$

$\sum i^3 = \left(\sum i\right)^2$ so $\sum_{i=1}^{20} i^3 = (210)^2 = k^2$ and $k=\boxed{210}$ since $k>0$.

6-4) $\frac{x^2-36}{x^2-9} = 1 - \frac{27}{x^2-9}$ is an integer if $x^2-9 = \pm1, \pm3, \pm9, \pm27$. Continuing, $x^2 = 0, 6, 8, 10, 12, 18, 36$, so $x = \boxed{0, \pm\sqrt{6}, \pm\sqrt{8}, \pm\sqrt{10}, \pm\sqrt{12}, \pm\sqrt{18}, \pm 6}$.

6-5) If the linear dimensions of two similar solids are in the ratio 2:1, their volumes are in the ratio $2^3:1^3 = 8$ to 1. Consequently, the capacity of the bigger container is $\boxed{8}$ liters.

6-6) Since $b-c = (a-c)-(a-b)$, we can write

$$\begin{aligned}
a^2(b-c)-b^2(a-c)+c^2(a-b) &= a^2[(a-c)-(a-b)] - b^2(a-c)+c^2(a-b) \\
&= a^2(a-c) - a^2(a-b) - b^2(a-c) + c^2(a-b) \\
&= (a-c)(a^2-b^2) + (a-b)(c^2-a^2) \\
&= (a-c)(a+b)(a-b) + (a-b)(c+a)(c-a) \\
&= (a-c)(a-b)(a+b-c-a) = \boxed{(a-b)(a-c)(b-c)}.
\end{aligned}$$