Bergen County Mathematics League

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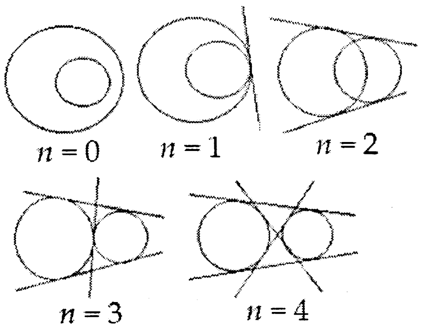
Brief Contest Solutions #2 **2015-2016**

2-1. For $a \neq 0$, $n^2 > n^2 - a^2 = (n + a)(n - a)$, thus $n^2 > (n + a)(n - a)$ holds for all nonzero a. We have $2015! = 1\times2\times3\times \dots \times1008\times \dots \times2013\times2014\times2015$. The middle term is 1008, so $1008^2 > 1007\times1009 > 1006\times1010 > \dots > 1\times2015$. Thus $\boxed{1008^{2015}} > 2015!$.

2-2.



The possible values of n are $\boxed{0, 1, 2, 3, 4}$.

2-3. We have $x^4 + 12x^3 + 36x^2 - 10 = (x^2 + 6x)^2 - 10 = 45^2 - 10 = \boxed{2015}$.

2-4. Let v m/s be the speed of the tortoise. We see that the speed of the hare was $5v$ m/s. The hare actually ran 9900m, and the tortoise actually ran 10000m. The match took $\frac{10000}{v}$ seconds (because the tortoise was running all the time). The hare ran for $\frac{9900}{5v}$ seconds and slept for the rest of the time. The hare slept for $\frac{10000}{v} - \frac{9900}{5v}$ seconds. During the time that the hare was sleeping, the tortoise ran $v\left(\frac{10000}{v} - \frac{9900}{5v}\right) = \boxed{8020}$m.

2-5. We have $AB = 5$. By area, we see that $AC\times BC = AB\times CD$, which is $3\times4 = 5CD$. Thus, $CD = 12/5$. Using the Pythagorean Theorem on $\triangle BCD$, we get $BD = 16/5$. In $\triangle BCD$, we have $BD\times CD = BC\times DE$, or $(16/5)(12/5) = 4DE$. Therefore, $DE = \boxed{48/25}$. [Note: Use similar right triangles for an alternate solution.]

2-6. Divide the problem into cases. Different letters represent different dishes:

1. 4 different dishes ($ABCD$): There are $\binom{10}{4} = 210$ possibilities.
2. Exactly 2 boxes have the same dish ($AABC$): There are $\binom{10}{3}\binom{3}{1} = 360$ possibilities.
3. 2 pairs of the same dishes ($AABB$): There are $\binom{10}{2} = 45$ possibilities.
4. Exactly 3 boxes have the same dish ($AAAB$): There are $\binom{10}{2}\binom{2}{1} = 90$ possibilities.
5. All of the boxes contain the same dish ($AAAA$): There are $\binom{10}{1} = 10$ possibilities.

Adding the possibilities from these 5 cases, there are a total of $\boxed{715}$ possibilities.