Bergen County Mathematics League

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Brief Contest Solutions #4 **2015-2016**

4-1. Each number in the second sequence is 63 less than the corresponding number in the first sequence. Thus the sum of the numbers in the first sequence is greater than the sum of the numbers in the second sequence by $63^2 = \boxed{3969}$. [Note: The sum of the integers from 64 to 126 is 5985. The sum of the integers from 1 to 63 is 2016.]

4-2. If the initial quantity is I, the result of increasing I by $p\%$ is $I(1+\frac{p}{100})$, and the result of decreasing I by $p\%$ is $I(1-\frac{p}{100})$. Here, $A = 70(1-\frac{20}{100}) = 56$, $B = 56(1+\frac{25}{100}) = 70$, $C = 70(1-\frac{60}{100}) = 28$, and $D = 28(1+\frac{150}{100}) = \boxed{70}$.

4-3. Since R's radius and height have respective lengths 5 and 8, R's volume is $\pi r^2 h = \pi(5)^2(8) = \boxed{200\pi}$.

4-4. Reading the given information in the reverse order, we get Figure 1. Then we proceed to get Figure 2. From Figure 2, we get the answer $3 + 4 + 5 + 6 + 7 + 8 + 9 = \boxed{42}$.

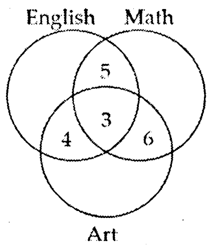


Figure 1

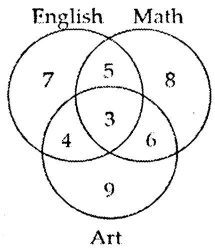
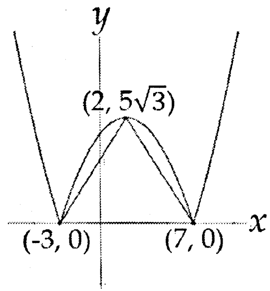


Figure 2

4-5. By factoring, we have $y = a|(x+3)(x-7)|$. The zeros are -3 and 7, and the vertex is at $x = 2$, which is the midpoint of the zeros. Each side of the equilateral triangle is 10. The distance from (-3, 0) to (2, y) is 10. By the distance formula, $y = 5\sqrt{3}$, so $(2, 5\sqrt{3})$ is the vertex. Since this lies on $y = a|x^2 - 4x - 21|$, substitute the coordinates of the vertex to get $a = \boxed{\frac{\sqrt{3}}{5}}$.



4-6. To establish that $a > b$, note that if $a = b$, then $b = c = d$, creating a contradiction that $a > d$; if $a < b$, then $b < c < d$, creating a contradiction that $a > d$. Similarly, we see that $b > c$ and $c > d$, from which we get $a > b > c > d$.

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k > 1$, from which $c = dk$, $b = dk^2$, and $a = dk^3$.

Now, $\frac{a-d}{b-c} = \frac{dk^3-d}{dk^2-dk} = \frac{k^3-1}{k^2-k} = \frac{(k-1)(k^2+k+1)}{k(k-1)} = \frac{k^2+k+1}{k}$.

As k approaches 1, the expression approaches $\boxed{3}$ from above.