Bergen County Mathematics League

Good Luck To You

Good Luck to All

Part I *Time Limit:* 12 *minutes* Answers must be exact or have 4 (or more) significant digits, correctly rounded.

4-1. Let N be an integer with all distinct nonzero digits such that $1000 \le N \le 9999$, and reading from left to right, each pair of consecutive digits of N is a multiple of 12. What is the largest possible value of N?

4-2. What is the only prime number p for which $2011p + 9$ is a perfect square? [Note: 2011 is a prime number.]

Part II *Time Limit:* 12 *minutes*

4-3. What are all ordered pairs of real numbers (x, y) that satisfy both $x^2 + xy = 77$ and $xy + y^2 = 44$?

4-4. What is the area of a trapezoid whose bases are 5 and 25 and whose diagonals are 18 and 24?

Part III *Time Limit:* 12 *minutes*

4-5. What is the smallest positive integer N for which $2N$ is a perfect square and $3N$ is a perfect cube?

4-6. If $\log_6 27 = a$, then, explicitly in terms of a, what is the value of $\log_{18} 16$?

Answers

4-1. 7248
4-2. 2017
4-3. (7, 4), (-7, -4)
4-4. 216
4-5. 72
4-6. $\frac{4(3-a)}{3+a}$