Bergen County Mathematics League

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1-1. By inspection, one solution is $x = 2016$. Since x and $\frac{1}{x}$ are reciprocals, a second solution will be $\frac{1}{2016}$. The two answers are **2016, 1/2016**.

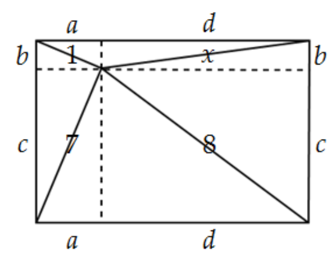
Alternatively, by clearing fractions we get $2016x^2 + 2016 = 2016^2x + x$, for which $2016x^2 - (2016^2 + 1)x + 2016 = 0$, or $(x - 2016)(2016x - 1) = 0$.

1-2. Clearly, 959 310 is a multiple of 5, which means one member must be 15 years old. Also, 959 310 is a multiple of 9 because the sum of its digits is divisible by 9, so one member must be 18 years old. Since $(9 + 9 + 1) - (5 + 3 + 0)$ is a multiple of 11, the number is divisible by 11, so one member must be 11 years old. Since $959310 \div 15 \div 18 \div 11 = 323$, and 323 = 17 ´ 19, the sum of the ages is $15 + 18 + 11 + 17 + 19 =$ **80**.

1-3. Let the price of the cheaper compass be x. The price of the other compass will be $1000 - x$. Therefore, $1.01x = 0.99(1000 - x)$, or $2x = 990$, from which $x =$ **495 or $495**.

1-4. Let A be the number of coins that Al found, B be the number of coins that Barb found, and C be the number of coins that Cal found. Since $A: B = 5: 4 = 25: 20$ and $B: C = 5: 6 = 20: 24$, it follows that $A: B: C = 25: 20: 24$. Solving $25x + 20x + 24x = 345$, we get $x = 5$ and $A =$ **125**.

1-5. In the diagram shown, using the Pythagorean Theorem, we get $a^2 + b^2 + c^2 + d^2 = 65$, and $a^2 + c^2 + b^2 + d^2 = 49 + x^2$. Therefore, $65 = 49 + x^2$, and $x =$ **4**.



1-6. Let $ab + a + b = n$. Adding 1 to both sides and factoring, we have $(a + 1)(b + 1) = n + 1$. Since a and b are both positive integers, each factor on the left must be at least 2. Therefore $n + 1$ cannot be a prime. The only two primes from 2011 to 2020 inclusive are 2011 and 2017 (The even numbers from 2011 to 2020 inclusive are not primes; 2013 and 2019 are not primes because they are divisible by 3; 2015 is not a prime because it is divisible by 5.), so n cannot be **2010, 2016**.