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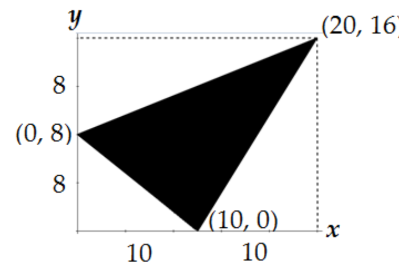
2-1. The remainder of 2016 ÷ 360 is 216, so turning the wheel 2016° counterclockwise is the same as turning it **216°** counterclockwise.

2-2. To find the area of the shaded triangle, subtract the sum of the areas of the three unshaded triangles from the large rectangle. The area of the rectangle is 20 ´ 16 = 320. The unshaded triangles have total areas 40 + 80 + 80 = 200, so the area of the shaded triangle is 320 – 200 = **120**.



2-3. We can express any number from 100 to 999 inclusive with $100A + 10B + C$, where A, B, and C are digits with $A \neq 0$. Since $(100A + 10B + C) - A - B - C = 99A + 9B = 9(11A + B)$, from which it must be divisible by **9**.

2-4. Let $a = x - 3k$ and $b = x - k$. It follows that $c = x + k$ and $d = x + 3k$. Since the sequence is increasing, $k > 0$. We have $a + b + c + d = 4x = 20$, from which $x = 5$. Since $\frac{ad}{bc} = \frac{2}{3}$, we get $\frac{(5-3k)(5+3k)}{(5-k)(5+k)} = \frac{25-9k^2}{25-k^2} = \frac{2}{3}$. Clearing fractions results $25 = 25k^2$, from which $k = 1$. Therefore, $a = 5 - 3(1) =$ **2**.

2-5. The product of the roots is 12. Since 12 = 1 ´ 12 = 2 ´ 6 = 3 ´ 4 = (-3) ´ (-4) = (-2) ´ (-6) = (-1) ´ (-12), the possible sums of the roots are 13, 8, 7, -7, -8, -13. Since the sum of the roots of $x^2 + ax + 12$ is $-a$, all possible values of a are **-13, -8, -7, 7, 8, 13**.

2-6. As shown, the radius of circle D is $DB + BE$. We know that $BE = 8$. What is DB? Extend $\overline{CD}$ so that it intersects $\overline{AB}$ at M. Since $\triangle ABC$ is equilateral, $\overline{CM}$ is a perpendicular bisector, an angle bisector, and an altitude. Also, $\overline{BD}$ is an angle bisector for $\angle ABC$. Since $\triangle MDB$ is a 30°-60°-90° triangle, $MB = 4$ and $DB = \frac{8}{\sqrt{3}}$. Therefore, the radius of circle D is $\boxed{8 + \frac{8}{\sqrt{3}}}$.