

## Bergen County Mathematics League

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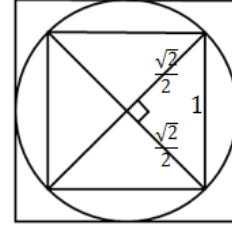


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- 3-1. Without loss of generality, let the side-length of the smaller square be 1. The diagonals of a square divide the shape into four isosceles right triangles. As shown in the diagram, the radius of the circle is  $\frac{\sqrt{2}}{2}$ . The side-length of the larger square is twice of that of the radius of the circle, from which it is  $\sqrt{2}$ . Since the area of the larger square is 2 and the area of the smaller square is 1, their ratio is  $\boxed{2: 1 \text{ or } 2}$ .



- 3-2. We have  $P(\text{GM wins}) = 60\%$  and  $P(\text{M does not win}) = 1 - 20\% = 80\%$ . Using conditional probability, the answer is  $\frac{P(\text{GM wins})}{P(\text{M does not win})} = \boxed{3/4 \text{ or } 0.75 \text{ or } 75\%}$ .
- 3-3. In the solution, define  $x R y$  to be the remainder when  $x$  is divided by  $y$ . If  $x < y$ , then  $x R y = 3$  implies that  $x = 3$ . However,  $y R x = 4$  is impossible since the remainder must be strictly less than the divisor. Therefore,  $x > y$ , from which  $y R x = 4$  implies that  $y = 4$ , and  $x R y = 3$  implies that the minimum value of  $x$  is 7. Finally, the minimum value of  $20x + 16y$  is  $20(7) + 16(4) = \boxed{204}$ .
- 3-4. Let the prices of a glove, a ball, and a helmet be  $x$ ,  $y$ , and  $z$  respectively. We know that  $21x + 21y = 28z$ . Both sides of equation represent the amount of money I have. We want to evaluate  $\frac{28z}{x+y+z}$ . Let's write everything in terms of  $z$ . From  $21x + 21y = 28z$ , we get  $x + y = \frac{4}{3}z$ . Therefore,  $\frac{28z}{x+y+z} = \frac{28z}{\frac{4}{3}z+z} = \frac{28z}{\frac{7}{3}z} = \boxed{12}$ .
- 3-5. If  $y = x^2 + 2x$ , then  $(x^2 + 2x - 1)(x^2 + 2x - 5) - 21 = (y - 1)(y - 5) - 21 = y^2 - 6y - 16 = (y - 8)(y + 2) = (x^2 + 2x - 8)(x^2 + 2x + 2) = \boxed{(x + 4)(x - 2)(x^2 + 2x + 2)}$ .
- 3-6. In the solution, define  $f_2(x) = f(f(x))$ ,  $f_3(x) = f(f(f(x)))$ , and so on. First note that  $f(17) = f_2(22) = f(19) = f_2(24) = f(21) = 18$ . Therefore,  $f_3(17) = f_2(18) = f_3(23) = f_2(20) = f(17) = 18$  as well. Finally,  $f(1) = f_2(6) = f_3(11) = f_4(16) = f_5(21) = f_4(18) = f_5(23) = f_4(20) = f_3(17) = \boxed{18}$ .