

## Bergen County Mathematics League

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6-1. Factoring the numerator of the right side, we get that  $2017! - 2016! = 2016!(2017 - 1) = 2016!(2016)$ . Divide this by 2016, the denominator of the right side, to get 2016!. Therefore,  $n! = 2016!$ , so  $n = \boxed{2016}$ .

6-2. If a positive integer has 2 positive divisors, then it must be a prime. We have 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47, a total of 15 numbers.  
If a positive integer has 3 positive divisors, then it must be the square of a prime. We have 4, 9, 25, and 49, a total of 4 numbers.  
If a positive integer has 5 positive divisors, then it must be the fourth power of a prime. We have 32, a total of 1 number.  
If a positive integer has 7 positive divisors, then it must be the sixth power of a prime. Since  $2^6 = 64 > 50$ , there is no number that satisfies the requirement.  
Together, we have a total of  $15 + 4 + 1 = \boxed{20}$  numbers.

6-3. Clearly,  $n$  is a positive integer. Since  $1.125n = \frac{9}{8}n$  and  $1.44n = \frac{36}{25}n$ , we want to "clear" the denominators. Therefore,  $n$  is the least common multiple of 8 and 25, which is  $\boxed{200}$ .

6-4. Let  $j$ ,  $s$ , and  $d$  be the speeds of Jerry, Steve, and Dan in laps/min, respectively. We have  $20j = 20s + 1$  and  $30j = 30d + 1$ . Multiplying the first equation by 3 and the second by 2, we get  $60j = 60s + 3$  and  $60j = 60d + 2$ . Subtracting the second equation from the first, we get  $0 = 60s - 60d + 1$ , from which  $60d - 60s = 1 = 60(d - s)$ . Therefore, in 60 minutes Dan will be 1 lap ahead of Steve. That's  $\boxed{30}$  minutes after the moment Jerry outpaces Dan by 1 lap.

6-5. Let  $P(x) = Ax^3 + Bx^2 + Cx + D$ . We get  $P(0) = D = 5$ ,  $P(1) = A + B + C + D$ ,  $P(-1) = -A + B - C + D$ ,  $P(2) = 8A + 4B + 2C + D$ , and  $P(-2) = -8A + 4B - 2C + D$ . We have  $P(1) + P(-1) = 2B + 2D = 32$ . Since  $D = 5$ , we conclude that  $B = 11$ . Therefore,  $P(2) + P(-2) = 8B + 2D = 8(11) + 2(5) = \boxed{98}$ .

6-6. Clearing fractions, we get  $c(b + c) + a(a + b) = (a + b)(b + c)$ , for which  $c^2 + a^2 - ac = b^2$ .  
By the Law of Cosines, we have  $\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2}$ . Therefore,  $m\angle B = \boxed{60 \text{ or } 60^\circ}$ .