

Bergen County Mathematics League

Good Luck To You



Good Luck To All

Contest #1 (No Calculators)

2017-2018

Part I *Time Limit: 12 minutes* On contests #2, #4 and #6, **any S.A.T. calculator** will be allowed.

1-1. If an author writes 2 extra pages each day, then she can complete her next book 3 days ahead of schedule—but if she writes 4 extra pages each day, then she can finish her next book 5 days ahead of schedule. What is the total number of pages in her next book?

1-2. For what ordered pair (a,b) is $\{x|a \leq x \leq b, x \neq 0\}$ all real solutions of $\frac{1 - \sqrt{1-2x^2}}{x} \leq 1$?

Part II *Time Limit: 12 minutes*

1-3. If \overline{AB} is a leg of the right triangle of least perimeter for which the lengths of the sides are integers, the length of the hypotenuse is 1 more than AB , and $AB > 100$, what is AB ?

1-4. What is the least positive integral remainder that results when 5 is divided into the sum $4^0 + 4^1 + 4^2 + 4^3 + 4^4 + 4^5 + 4^6 + 4^7 + 4^8 + 4^9 + 4^{10}$?

Part III *Time Limit: 12 minutes*

1-5. What is the sum of the first 100 terms of the sequence 1, 3, 3, 5, 5, . . . in which the first term is 1 and the $(2n)^{\text{th}}$ and $(2n+1)^{\text{st}}$ terms are both $2n+1$?

1-6. Two real numbers, each between 0 and 1, are chosen at random. What is the probability that the sum of their squares is greater than 1?

Answers

1-1. 120

1-2. $(-\sqrt{2}/2, 2/3)$

1-3. 112

1-4. 1

1-5. 5100

1-6. $1 - (\pi/4)$