Bergen County Mathematics League

Good Luck To You



Good Luck To All

Contest #3 (No Calculators)

2017-2018

Part I Time Limit: 12 minutes

On contests #4 and #6, any S.A.T. calculator will be allowed.

- 3-1. I went to the post office where I bought 40 stamps for \$1. Each stamp had a face value of 1¢, 4¢, or 12¢. If I bought at least 1 of each of these 3 types, how many 4¢ stamps did I buy?
- 3-2. In $\triangle ABC$, altitude \overline{CD} intersects median \overline{AM} at a point P in the interior of the triangle. If CD = 6 and AM = 5, what is the numerical value of the ratio AD:AP?

Part II Time Limit: 12 minutes

- 3-3. Bases \overline{AB} and \overline{CD} of trapezoid ABCD are each perpendicular to leg \overline{AD} . If CD = 6, AB = 8, and AD = 10, what is the average of the areas of all non-congruent triangles two of whose vertices are A and D, and whose third vertex is a point on \overline{BC} .
- 3-4. Three numbers are in arithmetic progression. The sum of the first and third numbers exceeds the second by 18. The sum of the squares of the first and third numbers exceeds twice the square of the second also by 18. What is the largest of these three numbers?

Part III Time Limit: 12 minutes

- 3-5. What are all two-digit numbers N for which the sum that I get when I add N to three times the sum of N's digits is a number with the same digits as N, but in reverse order?
- 3-6. If $\cos 36^\circ = (1+\sqrt{5})/4$, what is the value exact of $\cos 108^\circ$? [To get credit, you must write your answer as a reduced fraction with a rational denominator.]

Answers

3-1. 9

3-2. 4:5 or 4/5 or 0.8

3-3. 35

3-4. 21

3-5. 12, 24, 36, 48

3-6. $(1-\sqrt{5})/4$