Bergen County Mathematics League

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Brief Contest Solutions #1

2017-2018

1-1. Let the quota be x pages per day and the time limit, y days.
Then $(x+2)(y-3)=xy$ and $(x+4)(y-5)=xy$. Solve

1-2. If $x \geq 0$, $1-\sqrt{1-2x^2} \leq x$, or $\sqrt{1-2x^2} \geq 1-x$

$1-2x^2 \geq 1-2x+x^2$

$x \leq \frac{2}{3}$

If $x<0$, same result follows.

Since $\sqrt{1-2x^2} \geq 0$, $-\sqrt{2}/2 \leq x \leq \sqrt{2}/2$.

} Both conditions satisfied by $-\frac{\sqrt{2}}{2} \leq x \leq \frac{2}{3}$, $x \neq 0$

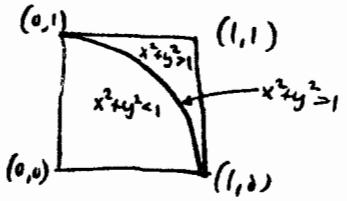
1-3. $a^2+b^2 = b^2+2b+1 \Rightarrow a^2 = 2b+1$.
So, looking for odd perfect square > 201.
Thus, least is $a^2 = 225$, $b = 112$.

1-4. $1 + \underbrace{(4+4^2)}_{\text{div. by 5}} + \underbrace{4^2(4+4^2)}_{\text{div. by 5}} + \underbrace{4^4(4+4^2)}_{\text{div by 5}} + \ldots + \underbrace{4^{48}(4+4^2)}_{\text{div by 5}}$.

1 ← remainder

1-5. $1+3=4$
$3+5=8$
$5+7=12$
$7+9=16$
$\vdots \quad \vdots$
} 50 terms

$\text{Sum} = 4(1+2+\ldots+50) = 4\left(\frac{50}{2}\right)(51)$
$= 5100$

1-6. 

$\text{Prob} = \frac{\text{square} - \text{quadrant}}{\text{square}} = \frac{1-\frac{\pi}{4}}{1}$