Bergen County Mathematics League

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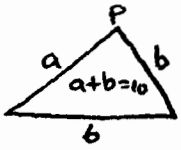
Brief Contest Solutions #2

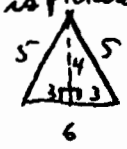
2017-2018

2-1. $\frac{x}{6}+\frac{x}{12}+\frac{x}{7}+5+\frac{x}{2}+4=x \Rightarrow x=84$, married at 21.

2-2. $x^4+4=(x^4+4x^2+4)-(4x^2)=(x^2+2)^2-(2x)^2=(x^2+2x+2)(x^2-2x+2)$
Who said you "can't" factor the sum of two squares?

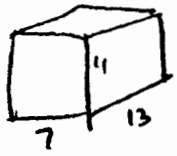
2-3. Locus of vertices P is an ellipse with foci 6 units apart and major axis = 10. Max area is pictured below.





2-4. $x<12\frac{1}{2} \Rightarrow \left[\frac{x}{12\frac{1}{2}}\right]=0=f(x)$
$x\geq 12\frac{1}{2} \Rightarrow \left[-\frac{12\frac{1}{2}}{x}\right]=-1.$
$\Big\} \therefore f(x)=-\left[\frac{x}{12\frac{1}{2}}\right]$ for all $x>0$.
Thus, $f(x)=-n$ whenever $12\frac{1}{2}n \leq x < 12\frac{1}{2}(n+1)$.

2-5. $1001 = 7\cdot 11\cdot 13$. Surface Area $= 2(7\cdot 11+11\cdot 13+7\cdot 13)=622$.



2-6. $a=\sqrt{x+y} \Rightarrow a>0$
$b=\sqrt{x-y} \Rightarrow b>0$
$\Big\}$ Thus $a^2+a-56=(a+8)(a-7)=0 \Rightarrow a=7$.
$b^2+b-30=(b+6)(b-5)=0 \Rightarrow b=5$.

$\therefore x+y=49$
$x-y=25$
$\Big\} \Rightarrow (x,y)=(37,12)$.