Bergen County Mathematics League

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Brief Contest Solutions #3

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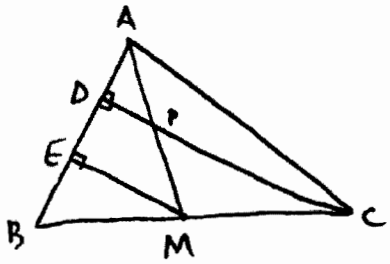
3-1. $\left.\begin{matrix} x+4y+12z=100 \\ x+y+z=40 \end{matrix}\right\} \Rightarrow 3y+11z=60$, or $y = 20 - 11\left(\frac{z}{3}\right)$,

Thus, $z = 0, 3$. (reject)

If $z=3$, $y=9$, $x=28$.

3-2.



Draw $\overline{ME} \parallel \overline{CD}$ making $ME = \frac{1}{2}(CD) = 3$.
In rt Δ MAE, $AE = 4$. By $\sim\Delta$s,
$AD : AP = AE : AM = 4:5$.

3-3. For all k, $0 < k \le 1$, for each vertex on $\overline{BC}$ with altitude $7+k$, there is another with altitude $7-k$. Thus, average area $= \frac{1}{2}(10)(7)$.

3-4. $\left.\begin{matrix} a-d,\ a,\ a+d \\ 2a = a+18 \Rightarrow a = 18 \end{matrix}\right\} (18-d)^2 + (18+d)^2 = 2\cdot 18^2 + 18 \Rightarrow d = \pm 3$

3-5. $3(t+4) + 10t + 4 = 10u + t \Rightarrow 2t = 4$.

3-6. $108^\circ = 180^\circ - 2(36^\circ)$, so $\cos 108^\circ = -\cos(2\cdot 36^\circ) = -(2\cos^2 36^\circ - 1)$

$= -\left(2\,\frac{6+2\sqrt{5}}{16} - 1\right)$

$= \frac{1-\sqrt{5}}{4}$.