Bergen County Mathematics League

Problem Author:
Steve Conrad
www.mathleague.com

Problem Editor:
Dan Flegler
www.mathleague.com

Brief Contest Solutions #4

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4-1. $65535 = 2^0 + 2^1 + 2^2 + \ldots + 2^{x-1} = 2^x - 1 \Rightarrow x = 16$.

4-2. $\frac{1}{\log_3 12} + \frac{1}{\log_4 12}$

Theorem: $\log_b a = \frac{\log_c a}{\log_c b}$. for all $a, b, c > 0$, $c \neq 1$.

$= \frac{\log 3}{\log 12} + \frac{\log 4}{\log 12} = \frac{\log 12}{\log 12} = 1$.

4-3. A $\perp$ gives a 5-12-13 Δ.

4-4. Multiply the product by $(2-1) = 1$.

The product is $(2^2-1)(2^2+1)(2^4+1)\cdot\ldots\cdot(2^{2^{n-1}}+1)$

$= (2^4-1)(2^4+1)(2^8+1)\cdot\ldots\cdot(2^{2^n}+1)$

$= \ldots$

$= 2^{2^n} - 1$.

4-5. Adding, $5x^2 = 5 \Rightarrow x = \pm 1$. Continue from there.

4-6. $2x + 2y = xy \Rightarrow x = \frac{2y}{y-2} = 2 + \frac{4}{y-2}$. Thus, $y = 3, 4, 6$

$x = 6, 4, 3$.