Bergen County Mathematics League

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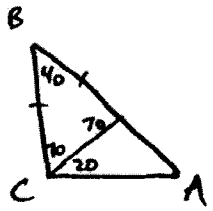
Brief Contest Solutions #5 **2017-2018**

5-1. $(x+1)^2 = x(x+2)+1$ is an identity.
So all the triples $(1,2,3)$, $(2,3,4)$, $(3,4,5)$, ..., $(97,98,99)$ work.

5-2. If $a+\frac{2}{a} = b+\frac{2}{b}$, $a=b$ or $a=\frac{2}{b}$.
Thus, $\sqrt{\frac{6x+6}{4x-1}} = 3$ or $\frac{2}{3}$. Solve.

5-3.



5-4. $x(x^2+11) = 6(x^2+1) \Rightarrow x^3-6x^2+11x-6=0$
$\Rightarrow (x-1)(x-2)(x-3)=0$.

5-5. Use $\sin A+\sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos A+\cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$.
to get $\frac{2\sin 30^\circ\cos(-10^\circ)}{2\cos 30^\circ\cos(-10^\circ)} = \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

5-6. Divisibility by 9 $\Rightarrow$ $3+A+B+8 = 18$ or 27 or ...
" " 11 $\Rightarrow$ $(A+8)-(B+3) = 0, 11, 22, \ldots$
or $A-B = -5, 6, 17, \ldots$
Adding, $2A = 2, 13, 24, \ldots$ or $A = 1$, only.
Thus, $B = 6$.