Bergen County Mathematics League

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Brief Contest Solutions #6

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6-1. $\sqrt{\sqrt{\sqrt{2}}} = \left(\left(2^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = 2^{\frac{1}{8}} = 2^{2^{-3}}$

$\log_2 2^{2^{-3}} = 2^{-3}$, $-\log_2 2^{-3} = 3$.

6-2. In 1 minute, minute hand moves $6°$. hour hand moves $\frac{1}{2}°$. } During x minutes, $\Rightarrow$ hands from $6x°$, $\frac{1}{2}x°$.

At 4 o'clock, $\measuredangle$ between hands $= 120°$. }

$\therefore \quad 6x - \frac{1}{2}x = 120.$

6-3. $4x - 4y = 36 \Rightarrow x - y = 9.$

$x^2 - y^2 = 243 \Rightarrow (x+y)(x-y) = 243 \Rightarrow x+y = 27.$ } $\therefore x = 18, y = 9.$

6-4. $x, x+2, x+4$. Since x is divisible by 5, so is $x-5$. }

$x+2-7$ is still divisible by 7. } each is $x-5$.

$x+4-9$ is still divisible by 9. }

Therefore, $x-5 = (5)(7)(9)(n)$ and $x = 315n+5$.

For $n=1$, $x = 320$. For $n=2$, x is odd. If $n > 2$, $x > 900$.

6-5. $\frac{a+24}{m} = \frac{2}{1} \Rightarrow 2m - 24 = a.$

$\frac{a}{m-x} = \frac{2m-24}{m-x} = \frac{2}{1} \Rightarrow x = 12.$

6-6. $1 - \cos A = 2\sin^2 \frac{1}{2}A$

$1 + \cos A = 2\cos^2 \frac{1}{2}A$

$\sin A = 2\sin \frac{1}{2}A \cos \frac{1}{2}A$ } $\therefore$ given $= \frac{2\sin^2 20° + 2\sin 20° \cos 20°}{2\cos^2 20° + 2\sin 20° \cos 20°} = \frac{\sin 20°}{\cos 20°}.$