Contest #5 Bergen County Math League 2018–2019

Part I *Time Limit:* 12 minutes **Calculators Allowed**

Answers must be exact or have 4 (or more) significant digits, correctly rounded.

5–1. A circular path is 285 meters in circumference. Starting at point P on the path, a man makes a mark every 60 meters, stopping only when he next makes a mark at P. Find the shortest (positive) distance (along the circular path) between two of the marks.

5–2. Find the smallest positive integer n for which $1350n$ is the cube of an integer.

Part II *Time Limit:* 12 minutes **Calculators Allowed**

Answers must be exact or have 4 (or more) significant digits, correctly rounded.

5–3. Find the ordered pair of real numbers (a, b) for which

$$(x^2 + ax + b)^2 = x^4 - 4x^3 + 10x^2 - 12x + 9.$$

5–4. How many points, all of whose coordinates are integers, are interior to the sphere $x^2 + y^2 + z^2 = 9$?

Part III *Time Limit:* 12 minutes **Calculators Allowed**

Answers must be exact or have 4 (or more) significant digits, correctly rounded.

5–5. If A and B are acute angles for which $\sin A + \sin B = \cos A + \cos B$, find the value of $A + B$.

5–6. Find the measure of the smaller angle formed by the hour and minute hands of a clock at 12:44.

Answers

5–1. 15 (meters)

5–2. 20

5–3. $(-2, 3)$

5–4. 93

5–5. 90° or $\frac{\pi}{2}$

5–6. 118 or 118°